ESIEvoaeje ovoke ucnv
18/ENG03/026
Civil Engineering.

1) Mathematical modelling is the process of setting up a model solving it mathematically and interpreting the results in physical or in other terms.

ii) Using balance law - law of conservation of mass
b) Forming a differential equation from an existing algebraic equation of the system.

2) $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, k + 6e^{2t} k$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$= 2i + 18\sin 3t\, j + 12e^{2t} k$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0) j + 12e^{2(0)} k$

$= 2i + 12k$

3) If

$A = x^2y\, i + (xy + yz)j + xz^2 k$

$B = yz\, i - 3xz\, j + 2xy k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

at point $(1, 2, 1)$

i) $\Delta\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z} k$

$\Delta\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 4y^2 z)k$

at point $(1, 2, 1)$

$\Delta\phi = (6(1)(2) + (2)(1))i + ((3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 4(2)^2(1))k$

$\Delta\phi = 14i - 12j - 16k$

ii) $\Delta A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot a_x i + a_y j + a_z k$

$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$= \frac{\partial x^2 y}{\partial x} + \frac{\partial (xy + yz)}{\partial y} + \frac{\partial}{\partial z}(xz^2)$

$= 2xy + (x + z) + x 2z$

$= 2(1)(2) + (1+1) + (1)2(1)$

$= 4 + 2 + 2$

at $(1, 2, 1)$

$\Delta A = 4 + 2 + 2$

$\Delta A = 8$

iii) $\Delta \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3x & 2xy \end{vmatrix}$

$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3x & 2xy \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3x \end{vmatrix}$

$i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right] + k\left[\frac{\partial}{\partial x}(-3x) - \frac{\partial}{\partial y}(yz)\right]$

$\Delta \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

at point $(1, 2, 1)$

$\Delta \times B = i(2(1) + 3(1)) - j[2(2) - (2)] + k[-3(1) - (1)]$

$= 5i - 2j - 4k$

iv) grad div A

$\text{div } A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot a_x i + a_y j + a_z k$

$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$= \frac{\partial (x^2 y)}{\partial x} + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial xz^2}{\partial z}$

$= 2xy + (x + z) + x 2z = A$

grad Div $A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\cdot(2xy + (x+z) + xz^2)$

$= (2y + 2 + 2z)i + j(2x) + (1+2)k$

at point $(1,2,1)$

grad Div $A = (2(2) + 2(1) + 2(1))i + (2(1))j + (1+2)k$

$\Delta A = 2i + 2j + 3k$

v) Curl / curl of

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+y^2) \end{vmatrix}$$

$i[0-y] - j[z^2 - 0] + k(y - x^2)$

$= -yi - z^2 j + (y - x^2)k$

$+k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$

$\text{Curl Curl } A = i[1+2z] - j[-2x+0] + k(0+1)$

$\text{Curl Curl } A = (1+2z)i + 2xj + k$

at point $(1,2,1)$

$\text{Curl Curl } A = (1+2(1))i + 2(1)j + k$

$= 3i + 2j + k$

(i) $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t(0)j + 12e^{2(0)}$

$t = 0 \quad 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$

$= 12.17$