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Question 1

(i) Mathematical Modelling is defined as the process of developing a mathematical model

(ii) Methods of developing model in engineering system
(i) Radioactivity
(ii) Mixing Problems

Question 2

~~If $r = (t^2+3t)i - 2\sin t j + 3e^{2t}k$~~

Soln

If $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t} k$

(i) $\frac{dr}{dt} = (2t+3)i - 2(3\cos 3t)j + 2(3e^{2t})k$

$= (2t+3)i - 6\cos 3t j + 6e^{2t} k$

(ii) $\frac{d^2r}{dt^2} = \frac{d(2t+3)i}{dt} - \frac{d}{dt}(6\cos 3t)j + \frac{d}{dt}(6e^{2t})k$

$\frac{d^2r}{dt^2} = 2i - (-(6\times 3)(\sin 3t)j) + 12e^{2t} k$

$= 2i + 18\sin 3t j + 12e^{2t} k$

Value of $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$= 2i + 18\sin 3(0) j + 12e^{2(0)} k$

$= 2i + 0j + 12k$

$= \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$= 2\sqrt{37}$ units

Question 3

$\nabla \phi$

When $\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$$\nabla \phi = i\frac{d\phi}{dx} + j\frac{d\phi}{dy} - \hat{k}\frac{d\phi}{dz}$$

$$\frac{d\phi}{dx} = 6xy + yz \quad \text{—(1)}$$

$$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$$

$$\frac{\partial\phi}{\partial z} = xy - 8y^2z$$

$$\nabla\phi = \hat{i}\frac{\partial\phi}{dx} + j\frac{d\phi}{dy} - \hat{k}\frac{\partial\phi}{dz}$$

$= \hat{i}(6xy + yz) + \hat{j}(3x^2 + xz - 8y^2z^2) + \hat{k}(xy - 8y^2z)$

at point $(1,2,1)$

$x = 1, y = 2$ and z

$= i(6(1)(2)(1) + (2)(1)) + j(3(1)^2 + (1)(1) - 8(2)(1)^2) + (1)(2) - 8(2)^2(1)\hat{k}$

$= (12 + 2)\hat{i} - 12j - 30\hat{k}$

$\therefore \nabla\phi = 14i - 12j - 30k$

(ii) $\nabla . A$

$$= \frac{\partial ax}{\partial x} + \frac{\partial ay}{\partial y} + \frac{\partial az}{\partial z}$$

$$\nabla = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k$$

$\Delta = x^2yi + (xy + yz)j + xz^2k$

$\Delta . A = 2xy + (x + z) + 2xz$

$\Delta . A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 2 \times 2 + 2 + 2$

$= 4 + 2 + 2$

$= \Delta . A = 8$

(ii) Curl of B $= \nabla \times B$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3x & 2y \end{vmatrix}$$

$$\nabla \times B = i\left(\frac{\partial 2xy}{\partial y} + \frac{\partial 3x^2}{\partial z}\right) - j\left(\frac{\partial 2xy}{\partial x} - \frac{\partial yz}{\partial z}\right) + k\left(-\frac{\partial 3x^2}{\partial x} - \frac{\partial yz}{\partial y}\right)$$

$= (2x + 3x)\hat{i} - (2y - y)\hat{j} + (-3z - 2)$

at $(1,2,1)$

$= (2(1) + 3(1))\hat{i} - \hat{j}(2(2) - 2) + \hat{k}(-3(1) - 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

Gradient of div A $= \nabla(\nabla \cdot A)$

$\nabla \cdot A = 2xy + (x + 2) + 2xz$

$\nabla(\nabla \cdot A) = \hat{i}\,\frac{\partial(\nabla \cdot A)}{\partial x} + \hat{j}\,\frac{\partial(\nabla A)}{\partial y} + \hat{k}\left(\frac{\partial \nabla A}{\partial z}\right)$

$\frac{\partial(\nabla \cdot A)}{\partial x} = (2y + 2 + 2z)\hat{i}$

$\frac{\partial(\nabla \cdot A)}{\partial y} = (2x + 0)\hat{j}$

$\frac{\partial(\nabla \cdot A)}{\partial z} = (x + 2x)\hat{k}$

$\nabla(\nabla \cdot A)$ at $(1,2,1)$

$\nabla(\nabla \cdot A) = (2(2) + 1 + 2(1))\hat{i} + 2(1)\hat{j} + (1 + 2(1))\hat{k}$

$= (4 + 1 + 2)\hat{i} + 2\hat{j} + 3k$

$\nabla(\nabla \cdot A = 7i + 2j + 3k$

Curl A $= \nabla \times A$ $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$

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$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & (xz^2 \end{vmatrix} - y\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & (xz^2) \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy+yz) \end{vmatrix}$$

$= i\left(\partial/\partial y\,(xz^2) - \partial/\partial z\,(xy+y^2)\right) - j\left(\partial/\partial x\,(xz^2) - \partial/\partial z\,(x^2 y)\right)$

$+ k\left(\partial/\partial y\,(xy+y^2) + \partial/\partial y\,(x^2 y)\right]$

$= i(0-y] - j(z^2 - 0] + k[y - x^2]$

$\nabla \times A = -yi - z^2 j + (y + x^2)\hat{k}$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2 \end{vmatrix}$$

$= i\left[\frac{\partial}{\partial y}(y - x^2) - \frac{d}{dz}(-z)^2\right] - j\left(\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right]$

$+ k\left[\frac{d}{dz}(-z^2) - \frac{d}{dy}(-y)\right]$

$= I(1+2z) - j(2x+1)) + k(0+1)$

$\nabla \times (\nabla \times \Delta$ at $(1,2,1)$

$\nabla \times (\nabla \times A) = i(1 \times 2 \times 1) - j(-2 \times 1) + k(1)$

$\nabla \times (\nabla \times A) = 3i + 2j + k$