$$\text{grad}\cdot(\text{div}A) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left(2xy + (x+z) + 2xz\right)$$

$$= i\frac{\partial}{\partial x}(2xy + x + z + 2xz) + j\frac{\partial}{\partial y}(2xy + x + z + 2xz) + k\frac{\partial}{\partial z}(2xy + x + z + 2xz)$$

$$= (2y + 1 + 2z)i + 2xj + (1 + 2x)k$$

at point $(1, 2, 1)$

$$\text{grad}\cdot\text{div}A = (2(2) + 1 + 2(1)) + (2(1))j + (1 + 2(1))k$$

$$\text{grad}\cdot\text{div}A = 7i + 2j + 3k$$

v. Curl Curl A

$$A = (x^2y)i + (xy + yz)k + (xz^2)k$$

$$\text{Curl}A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right] - j\left[\frac{\partial}{\partial x}xz^2 - \frac{\partial}{\partial z}x^2y\right] + k\left[\frac{\partial}{\partial x}xy+yz - \frac{\partial}{\partial y}(x^2y)\right]$$

$$= i[0 - y] - j[z^2 - 0] + k[y - x^2]$$

$$= -yi - z^2j + (y - x^2)k$$

$$\text{Curl}(\text{Curl}A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y - x^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y - x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y - x^2) - \frac{\partial}{\partial z}(-y)\right] + k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$$

$$\text{Curl Curl}A = i[1 + 2z] - j[-2x + 0] + k[0 + 1]$$

$$\text{Curl Curl}A = (1 + 2z)i + 2xj + k$$

at point $(1, 2, 1)$

$\nabla\phi = 14i - 12j - 30k$

ii $\nabla \cdot A = \frac{\delta a_x}{\delta x} + \frac{\delta a_y}{\delta y} + \frac{\delta a_z}{\delta z}$ (div A)

$A = (x^2y)i + (xy + yz)j + (xz^2)k$

$(a_x + a_y + a_z)$

$= 2xy + x + z + 2xz$

at point $(\overset{x}{1}, \overset{y}{2}, \overset{z}{1})$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$\nabla \cdot A = 4 + 2 + 2$

$\nabla \cdot A = 8$

iii $\nabla \times B = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ yz & -3xz & 2xy \end{vmatrix}$

$i\left[\frac{\delta}{\delta y}(2xy) - \frac{\delta}{\delta z}(-3xz)\right] - j\left[\frac{\partial}{\partial x} 2xy - \frac{\partial}{\partial z} yz\right] + k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$

$\nabla \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

$= 5xi - yj - 4zk$

at point $(1, 2, 1)$

$\nabla \times B = 5(1)i - 2j - 4(1)k$

$= 5i - 2j - 4k$

iv grad div A

$A = (x^2y)i + (xy + yz)j + (xz^2)k$

$\text{div} A = \frac{\partial}{\delta x} x^2y + \frac{\partial}{\partial y} xy + yz + \frac{\partial}{\partial z} xz^2 k$

$\text{div} A = 2xy + x + z + 2xz$

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ENG 282

Electrical/Electronics Engineering

1i Model is a miniature representation of something

Mathematical model is a representation in mathematical terms of the behaviour of real devices and objects.

ii Dimensional homogeneity

Conservation and balance principles

2. $r = (t^2 + 3t)i - (2\sin 3t)j + 3e^{2t}k$

i $\frac{dr}{dt} = (2t + 3)i - (6\cos 3t) + 6e^{2t}k$

ii $\frac{d^2r}{dt^2} = 2i + (18\sin 3t)j + 12e^{2t}k$

iii $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$

$= 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 0j + 12k$

$= 2i + 12k$

$= \sqrt{2^2 + 12^2}$

$= \sqrt{148} = 2\sqrt{37} = 12.17$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$ unit

3 $A = (x^2y)i + (xy + yz)j + xz^2k$

$B = yzi + -3xzj + 2xyk$ at point $(1, 2, 1)$ (x, y, z)

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i $\nabla\phi = \frac{\delta\phi}{\delta x}i + \frac{\delta\phi}{\delta y}j + \frac{\delta\phi}{\delta z}k$