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16/ENG07/008
Petroleum Engineering
ENG 282

1) Mathematical modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical or in order terms.

b) i, Law of conservation of mass (Balance law)

ii, Forming a differential equation from an existing algebraic equation of the system

2. $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t} k$

i, $\frac{dr}{dt} = (2t+3)i - 6\cos 3t + 6e^{2t} k$

ii, $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k$

iii, $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18(\sin 3(0))j + 12e^{2(0)}k$

$= 2i + 12k$

iv $\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2} = \sqrt{4+144}$

$= \sqrt{148}$

3. $A = x^2yi + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

at point $(1,2,1)$;

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i, $\nabla\phi = \frac{d\phi}{dx}i + \frac{d\phi}{dy}j + \frac{d\phi}{dz}k$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

at $x=1, y=2, z=1$;

$\nabla\phi = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2 1)k$

$\nabla\phi = 14i - 12j - 30k$

ii, $\nabla \cdot A = \frac{\partial dx}{\partial x} + \frac{\partial dy}{\partial y} + \frac{\partial dz}{\partial z}$

$\nabla \cdot A = 2xy + (x+z) + (2xz)$

at $x=1, y=2, z=1$

$\nabla A = 2(1)(2) + (1+1) + (2(1)(1))$

$\nabla \cdot A = 4+2+2$

$\nabla A = 8$

iii, $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(2xy) - 3xz\left(\frac{\partial}{\partial z}\right)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]$

$+ k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$

$\nabla \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

$\nabla \times B = 5xi - yj - 4zk$

at $x=1, y=2, z=1$

$\nabla \times B = 5(1)i - y(2)j - 4(1)k$

$\nabla \times B = 5i - 2j - 4k$

iv, grad·div $A = \nabla \cdot (\nabla A) = \nabla^2 A$

$= \frac{\partial}{\partial x}\frac{\partial A}{\partial x} + \frac{\partial}{\partial y}\frac{\partial A}{\partial y} + \frac{\partial}{\partial z}\frac{\partial A}{\partial z}$

$A = x^2y\,i + (xy + yz)j + xz^2k$

div $A = 2xy + (x+z) + 2xz$

grad·div $A = (2y + 1 + 2z)i + (2x)j + (1+2x)k$

at $x=1, y=2, z=1$

grad·div $A = (2(2) + 1 + 2(1))i + 2(1)j + (1+2(1))k$

grad·div $A = 7i + 2j + 3k$

iii, Curl Curl A $= \cancel{\nabla^2 \times A}\ \nabla \times (\nabla \times A)$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$\nabla \times A = [\partial/\partial y\,(xz^2) - \partial/\partial z\,(xy+yz)] - j[\partial/\partial x\,(x^2z) - \partial/\partial z\,(x^2y)]$$
$$+k\,[\partial/\partial x\,(xy+yz) - \partial/\partial y\,(x^2y)]$$

$$\nabla \times A = i[0-y] - j[z^2-0] + k(y-x^2)$$
$$\nabla \times A = -yi - z^2j + (y-x^2)k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$\text{Curl curl } A = i[\partial/\partial y\,(y-x^2) - \partial/\partial z\,(-z^2)] - j[\partial/\partial x\,(y-x^2) - \partial/\partial z\,(-y)]$$
$$+k[\partial/\partial x\,(-z^2) - \partial/\partial y\,(-y)]$$

$$\text{Curl curl } A = i[1+2z] - j[-2x+0] + k[0+1]$$
$$\quad " \quad = (1+2z)i + 2xj + k$$

at $x=1,\ y=2,\ z=1$

$$\text{Curl curl } A = (1+2(1))i + 2(1)j + k$$
$$\text{Curl curl } A = 3i + 2j + k$$

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