Assignment

1(a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and Interpreting the result in physical and other terms.

b i) Exponentional Growth Decay (use of ODE)
ii) Mixing problems.

2 $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$

i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t} k$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k$

iii) $\left.\frac{d^2r}{\partial t^2}\right|_{t=0} = 2i + 12k$

$$\left|\frac{\partial^2 r}{\partial t^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37}$$

$= 12.17$

3) $A = x^2 y i + (xy + y^2) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\theta = 3x^2 y + xyz - 4y^2 z^2 - 3$

i) $\nabla \phi = \frac{\partial \theta}{\partial x} i + \frac{\partial \theta}{dy} j + \frac{\partial \theta}{dz} k$

$\frac{\partial \theta}{\partial x} = 6xy + yz$

$$\frac{d\phi}{dz} = xy - 8y^2z$$

$$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$$

At $(1, 2, 1)$

$$\frac{\partial\phi}{dx} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\partial\phi}{dx} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$$

$$\frac{d\phi}{dz} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$$

$$\nabla\phi = 14i - 12j - 30k$$

ii) $\nabla \cdot A = \frac{da}{dx} + \frac{day}{dy} + \frac{daz}{dz}$

$A = axi + ayj + azk$

$\nabla \cdot A = 2xi + (x+1) + 2xz$

At $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/dx & \partial/dy & \partial/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At $(1, 2, 1)$

$\nabla \times B = 5i - 2j - 4k$

iv grad div A

grad $(2xy + (x+z) + 2xz)$

let $divA = c = \nabla A$

$$\nabla(\nabla A) = \nabla c = i\frac{dc}{dx} + j\frac{\partial c}{dy} + k\frac{dc}{\partial z}$$

$= i\,(2y + 1 + 2z) + j\,(2x) + k\,(1 + 2x)$

At $(1, 2, 1)$

$\bar{\nabla} c = i\,[2(2) + 1 + 2(1)] + j\,[2(1)] + k\,[1 + (2)(1)]$

$= i\,(4 + 1 + 2) + j\,(2) + k\,(1 + 2)$

$= 2i + 2j + 3k$

v) Curl A

Curl $A = \bar{\nabla} \times A$

$$= \begin{vmatrix} i & j & k \\ d/dk & d/dk & d/dz \\ x^2 y & (xy + yz) & 2z^2 \end{vmatrix}$$

$= i\,(0 - y) - j\,(z^2 - 0) + k\,(y - x^2)$

$= yi - z^2 j + k\,(y - x^2)$

At $(1, 2, 1)$

Curl $A = -2i - j + k$

Curl $A = \bar{\nabla} \times (\bar{\nabla} \times A)$

$$\bar{\nabla} \times (\bar{\nabla} \times A) \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$= i\,(1 + 2z) - j\,(-2x - 0) + k\,(0 + 1)$

$= i\,(1 + 2z) + 2x^2 j + k$

At point $(1, 2, 1)$

$\bar{\nabla} \times (\bar{\nabla} \times A) = i\,(1 + 2(1)) + 2(1)^2 j + k$

$= 3i + 2j + k$