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MECHANICAL ENGINEERING
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ENG 282

1(i) Mathematical modelling can be defined as the process of setting up a model of an engineering problem, solving it mathematically and interpreting the result in physical or other terms.
This model is the ~~formation~~ formulation of the problem as a mathematical expression in terms of variables, functions and equations.

ii Two methods of obtaining models for engineering systems
a using balance law
b forming a differential equation from an existing algebraic equation of the system

No 2

$r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

i $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, j + 6e^{2t} k$

ii $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k$

iii at $t = 0$, $\frac{d^2r}{dt^2} = 2i + 18\sin(3 \times 0) j + 12e^{2\cos} k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2 + (12)^2}$

$= \sqrt{4 + 144}$

$= \sqrt{148}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$

No 3

$A = x^3 yi + (xy + yz)j + xz^2 k$

$B = yzi - 3xz\,j + 2xy\,k$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

at the point $(1,2,1)$

(i) $\nabla\phi = \text{grad}\,\phi = \left(\frac{di}{dx} + \frac{dj}{dy} + \frac{dk}{dz}\right) \cdot (3x^2 y + xyz - 4y^2 z^2 - 3)$

$= \frac{d}{dx}(3x^2 y + xyz - 4y^2 z^2 - 3)i + \frac{d}{dy}(3x^2 y + xyz - 4y^2 z^2 - 3)j$

$+ \frac{d}{dz}(3x^2 y + xyz - 4y^2 z^2 - 3)k$

$= i(6xy + yz) + j(3x^2 + xz - 8yz^2) + k(8y - 8y^2 z)$

at the point $(1,2,1)$

$\nabla\phi = i[6(1)(2) + (2)(1)] + j[3(1)^2 + (1)(1) - 8(2)(1)^2] + k[(1)(2) - 8(2)^2(1)]$

$= i(12 + 2) + j(3 + 1 - 16) + k(2 - 32)$

$= 14i - 12j - 30k$

(ii) $\text{div}\,A = \nabla \cdot A = \left(\frac{di}{dx} + \frac{dj}{dy} + \frac{dk}{dz}\right) \cdot (x^2 yi + (xy + yz)j + xz^2 k)$

$\nabla \cdot A = \frac{d}{dx}(x^2 y) + \frac{d}{dy}(xy + yz) + \frac{d}{dz}(xz^2)$

$= 2xy + (x + z) + 2xz$

at $(1,2,1)$

$= 2(1)(2) + (1+1) + 2(1)(1) = 4 + 2 + 3$

$\nabla \cdot A = 8$

(iii) $\text{Curl}\,B = \nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$\nabla \times B = i\left[\frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]$

$+ k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$

$= i(2x + 3x) - j(2y - y) + k(-3z - 2)$
$= 5xi - yj - 4zk$

at $(1,2,1)$

$\nabla \times B = 5(1)i - (2)j - 4(1)k$
$= 5i - 2j - 4k$

iv grad div $A = \nabla(\nabla \cdot A)$

$\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot x^2yi + (xy + yz)j + xz^2k$

$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$\nabla(\nabla \cdot A) = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot 2xy + (x + z) + 2xz$

$= \frac{\partial}{\partial x}(2xy + (x+z) + 2xz)i + \frac{\partial}{\partial y}(2xy + (x+z) + 2xz)$

$+ \frac{\partial}{\partial z}(2xy + (x + z) + (2xz))k$

$= i(2y + 1 + 2z) + (2xj + (1 + 2x)k)$

at $(1,2,1)$

$\nabla(\nabla \cdot A) = [(2)\times(2) + 1 + 2(1)]i + 2(1)j + [1 + 2(1)]k$
$= [4 + 1 + 2]i + 2j + (1 + 2)k$
$= 7i + 2j + 3k$

v Curl $A = \nabla \times (\nabla \times A)$

$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy + yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right]$

$+ k\left[\frac{\partial}{\partial x}(xy + yz) - \frac{\partial}{\partial y}(x^2y)\right]$

$\nabla \times A = i(-y) - j(z^2)k + k(y - x^2$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y-x^2) + \frac{\partial}{\partial z}(z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) + \frac{\partial}{\partial z}(y)\right]$$

$$+ k\left[\frac{-\partial}{\partial x}(z^2) + \frac{\partial}{\partial y}(y)\right]$$

$$\nabla \times (\nabla \times A) = i(1 + 2z) - j(-2x) + k(1)$$

$$= \text{at } (1,2,1)$$

$$= i(1+2(1)) - j(-2(1)) + k$$

$$= i(1+2) - j(-2) + k$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k$$