UWALAKA EMEKA GODSWILL
16/ENG05/033
MECHATRONICS
ENG 282

1) Mathematical modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical or in order terms.

b) i, Law of conservation of mass (Balanced law)
ii, Forming a differential equation from an existing algebraic equation of the system.

2. $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$

i, $\frac{d^2r}{dt^2} = 2i + 18\sin 3t + 6e^{2t}k$

ii, $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

iii, $\left.\frac{d^2r}{dt}\right|_{t=0} = 2i + 18(\sin 3(0))j + 12e^{2(0)}k$
$= 2i + 12k$

iv, $\left|\frac{d^2r}{dt^2}\right| = \sqrt{(2)^2 + (12)^2} = \sqrt{4+144} = \sqrt{148}$

3. $A = x^2yi + (xy + yz)j + xz^2k$
$B = yzi - 3xzj + 2xyk$
at point $(1,2,1)$
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i, $\nabla\phi = \frac{d\phi}{dx} + \frac{d\phi}{dy}j + \frac{d\phi}{dz}k$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$
at $x=1,\ y=2,\ z=1;$

$\nabla\phi = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j$
$+ ((1)(2) - 8(2)(1))k$
$\nabla\phi = 14i - 12j - 30k$

ii, $\nabla \cdot A = \frac{d \cdot dx}{dx} + \frac{d \cdot dy}{dy} + \frac{d \cdot dz}{dz}$

$\nabla \cdot A = 2xy + (x+z) + (2xz)$

at $x = 1, \; y = 2, \; z = 1$

$\nabla A = 2(1)(2) + (1+1) + (2(1)(1))$

$\nabla \cdot A = 4+2+2$

$\nabla A = 8$

iii] $\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\left[\frac{d}{dy}(2xy) - 3x\left(\frac{d}{dz}\right)\right] - j\left[\frac{d}{dx}(2xy) - \frac{d}{dz}(y^2)\right]$

$+ k\left[\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right]$

$\nabla \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

$\nabla \times B = 5xi - yj - 4zk$

at $x = 1, \; y = 2, \; z = 1$

$\nabla \times B = 5(1)i - y(2)j - 4(1)k$

$\nabla \times B = \cancel{8(1)i} + 5i - 2j - 4k$

iv. $\text{grad} \cdot \text{div} A = \nabla \cdot (\nabla A) = \nabla^2 A$

$= \frac{d}{dx}\frac{dA}{dx} + \frac{d}{dy}\frac{dA}{dy} + \frac{d}{dz}\frac{dA}{dz}$

$A = x^2yi + (xy + yz)j + xz^2k$

$\text{div} A = 2xy + (x+z) + 2xz$

$\text{grad} \cdot \text{div} A = (2y + 1 + 2z)i + (2x)j + \cancel{(1+2x)}\,(1+2x)k$

at $x=1,\ y=2,\ z=1$

$\text{grad}\cdot\text{div}A = (2(2)) + 1+2(1))i + 2(1)j + (1+2(1))k$

$\text{grad}\cdot\text{div}A = 7i + 2j + 3k$

iii, Curl curl A $= \nabla \times (\nabla \times A)$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$\nabla \times A = \left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right] - j\left[\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right] + k\left[\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right]$$

$$\nabla \times A = i\,[0-y] - j\,[z^2-0] + k\,(y-x^2)$$

$$\nabla \times A = -yi - z^2 j + (y-x^2)k$$

$$\text{Curl curl } A = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$\text{Curl curl } A = i\left[\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right] - j\left[\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right] + k\left[\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right]$$

$$\text{Curl curl } A = i\,[1+2z] - j\,[-2x+0] + k\,[0+1]$$

$$\text{''} = (1+2z)i + 2xj + k$$

at $x=1,\ y=2,\ z=1$

$$\text{Curl curl } A = (1+2(1))i + 2(1)j + k$$

$$\text{Curl curl } A = 3i + 2j + k$$