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ENG 282 Assignment

(1)

(a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

b. i. Exponential growth (use of ODE)

ii. Mixing problems

(2) $r = (t^2+3t) - 2\sin 3t\,j + 3e^{2t}k$

i. $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

ii. $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$

iii. $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = \sqrt{4+144}$

$= \sqrt{148}$

$= 2\sqrt{37} = 12.7$

(3) $A = x^2y\,i + (xy+y^2)\,j + xz^2k$

$B = yz\,i - 3x\,z\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z - 3$

$\nabla\phi = \frac{\partial d_1}{\partial x} + \frac{\partial d}{\partial y}\,j + \frac{\partial dk}{\partial x}$

$\frac{\partial\phi}{\partial x} = 6xy + y^2$

$\frac{\partial \phi}{\partial z} = xy - 8y^2 z$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At (1,2,1)

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) - 2 - 32 = -30$

$\nabla \phi = 14i - 12j - 30k$

(ii) $\nabla \cdot A = \frac{da}{dx} + \frac{day}{dy} + \frac{daz}{dz}$

$A = axi + ayj + azk$

$\nabla \cdot A = 2xy + (x+2) + 2xz$

At (1, 1, 1)

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$4 + 2 + 2 = 8$

(iv) $\nabla \cdot B$

$$\begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At (1, 2, 1)

$\nabla \times B = 5i - 2j - 4k$

(iv) grad div A

grad $(2xy+(x+z)+2xz)$

let $\text{div}A = C = \bar{\nabla}A$

$\bar{\nabla}(\bar{\nabla}A) = \bar{\nabla}C = i\frac{dc}{dx} + j\frac{dc}{dy} + k\frac{dc}{dz}$

$= i(2y+1+2z) + j(2x) + k(1+2x)$

At $(1,2,1)$

$\bar{\nabla}c = i(x_i) + i + 2(1)) + j(2(1)) + k(1+(2)(1))$

$i(4+2) + j(2) + k(1+2)$

$= 7i + 2j + 3k$

(v) Curl Curl A

$\text{Curl}\, A = \bar{\nabla} \times A$

$$= \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ xy & (xy+yz) & xz^2 \end{vmatrix}$$

$= i(0-y) - j(z^2 - 0) + k(y - x^2)$

$= -y\,i - z^2 j + k(y - x^2)$

At $(1,2,1)$

$\text{Curl}\, A = -2i - j + k$, $\text{Curl Curl}\, A = \bar{\nabla} \times (\bar{\nabla} \times A)$

$$\bar{\nabla} \cdot (\bar{\nabla} \times A) \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$= i(1+2z) - j(-2x - 0) + k(0+1)$

$= i(1+2x) + 2x^2 j + k$, At Point $(1,2,1)$

$\nabla^2 \times (\bar{\nabla} \times A) = i(1+2(1)) + 2(1)^2 j + k$

$= 3i + 2j + k$