OYEWALE SEMILORE
MECHANICAL ENGINEERING
16/ENG 06/065.
ENGINEERING MATHEMATICS.

QUESTION 1

(i) Define Mathematical Modelling:

Mathematical Modelling is the art of translating problems from an application area into tractable Mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application

(ii) Outline two methods of obtaining models for engineering system

(1) By using the Balance law ie the law of conservation of mass

(2) A model can be formed by differentiating an existing algebraic equation of the system

QUESTION 2.

If

$$r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k.$$

determine

(i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t}k$

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

(iii) the Value of $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$.

$$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$$

$$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 12k$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{4+144}$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{148}$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$$

QUESTION 3

$A = x^2y\,i + (xy + yz)j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

determine, at the point (1, 2, 1)

(1) $\nabla\phi$

$$\nabla\phi = \frac{\delta\phi}{\delta x}i + \frac{\delta\phi}{\delta y}j + \frac{\delta\phi}{\delta z}k$$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$$\frac{\delta\phi}{\delta x} = 6xy + yz$$

$\frac{\delta\phi}{\delta z} = xy - 8y^2z$

at point (1,2,1)

$\frac{\delta\phi}{\partial x} = 6(1)(2) + (2)(1) = 14$

$\frac{\delta\phi}{\delta y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = -12$

$\frac{\delta\phi}{\delta z} = (1)(2) - 8(2)^2(1) = -30$

$\therefore \nabla\phi = 14i - 12j - 30k$ //

(ii) $\overline{\nabla} \cdot A$

$A = x^2 y i + (xy + yz) j + xz^2 k$

$\overline{\nabla} \cdot A = \left[i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right] (a_x i + a_y j + a_z k)$

$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$= 2xy + (x + z) + 2xz$

$= 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2$

$= 8$ //

(iii) $\overline{\nabla} \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \end{vmatrix}$

$$\bar{\nabla}\times B = i\begin{vmatrix} \delta/\delta y & \delta/\delta z \\ -3xz & 2xy \end{vmatrix} + j\begin{vmatrix} \delta/\delta x & \delta/\delta z \\ yz & 2xy \end{vmatrix}$$

$$+ k\begin{vmatrix} \delta/\delta x & \delta/\delta y \\ yz & -3xz \end{vmatrix}$$

$$= i\left(\frac{\delta 2xy}{\delta y} + \frac{\delta 3xz}{\delta z}\right) + j\left(\frac{\delta 2xy}{\delta x} - \frac{\delta yz}{\delta z}\right)$$

$$+ k\left(-\frac{\delta 3xz}{\delta x} - \frac{\delta yz}{\delta y}\right)$$

$$= i(2x + 3x) + j(2y - y) + k(-3z - z)$$

$$= 5xi + yj - 4zk$$

at point (1, 2, 1)

$$\bar{\nabla}\times B = 5(1)i + 2j - 4(1)k$$

$$= 5i + 2j - 4k$$

(iv) grad div A $= \bar{\nabla}(\bar{\nabla}\cdot A)$

$$\bar{\nabla}\cdot A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right](a_x i + a_y j + a_z k)$$

$$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$A = x^2 y i + (xy + yz)j + xz^2 k$$

$$\bar{\nabla}\cdot A = 2xy + (x + z) + 2xz$$

~~at (1, 2, 1)~~

$$\bar{\nabla}(\bar{\nabla}\cdot A) = \left[i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z} \right](2xy + (x+z) + 2xz)$$

$$\bar{\nabla}(\bar{\nabla}\cdot A) = i\frac{\delta A}{\delta x} + j\frac{\delta A}{\delta y} + k\frac{\delta A}{\delta z}$$

$$= i(2y + 1 + 2z) + j(2x) + k((0+1) + 2x)$$

at point $(1,2,1)$

$$= i(2(2) + 1 + 2(1)) + j(2(1)) + k((0+1) + 2(1))$$

$$= i(7) + j(2) + k(3)$$

$$\bar{\nabla}(\bar{\nabla}\cdot A) = 7i + 2j + 3k$$

v Curl curl A

curl $A = \bar{\nabla} \times A$

$$\bar{\nabla} \times A = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left(\frac{\delta xz^2}{\delta y} - \frac{\delta(xy+yz)}{\delta z}\right) - j\left(\frac{\delta xz^2}{\delta x} - \frac{\delta x^2 y}{\delta z}\right)$$

$$+ k\left(\frac{\delta(xy+yz)}{\delta x} - \frac{\delta x^2 y}{\delta y}\right)$$

$$= i(-y) - j(z^2) + k(y - x^2)$$

~~$= (-y)i + k(y - x^2)$~~

$= -yi - z^2 j + (y - x^2) k$

$\therefore$ Curl curl $A = \bar{\nabla}(\bar{\nabla} \times A)$

$$\bar{\nabla}(\bar{\nabla} \times A) = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

~~$= i \frac{\delta(y-x^2)}{\delta x}$~~

$$= i\left(\frac{\delta(y-x^2)}{\delta y} + \frac{\delta z^2}{\delta z}\right) + (-j)\left(\frac{\delta(y-x^2)}{\delta x} + \frac{\delta y}{\delta z}\right)$$

$$+ k\left(\frac{\delta - z^2}{\delta x} + \frac{\delta y}{\delta y}\right)$$

$= i(1 + 2z) - j(-2x) + k(1)$

$= i(1 + 2(1)) - j(-2(1)) + k(1)$

$= 3i + 2j + k$