MUFUTAU YUSUF ADEMOLA
16/ENG06/039 MECHANICAL ENG ENG282

(1) mathematically modelling is the process of setting up a model solving it mathematically and interpreting the result in physical or another term.

(ii) a) Using Balance law -> law of conversion of mass

(b) Forming a differential equations from an existing algebraic equation of the system

(2) $r = (t^2 + 3t)i - 2\sin 3t + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t + 6e^{2t}k$

(ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$= 2i + 18\sin t j + 12e^{2t}k$

(iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 12k$

3) if $A = x^2yi + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

determine at point (1, 2, 1)

1) $\nabla\phi = \frac{d\phi}{dx}i + \frac{d\phi}{dy}j + \frac{d\phi}{dz}k$

$(3x^2+xz-8yz^2)j+(xy-4y^2z)$ at point $(1,2,1)$ 14

$\nabla\phi=(6xy+yz)i+(3x^2+xz-8yz^2)j+(xy-4y^2z)$

$\nabla\phi=(6(1)(2)+(2)(1))i+((3(1)^2+(1)(1)-8(2)(1)^2+(1)(1)-8(2)(1)^2)j$

$+((1)(2)-4(2)2(1))k$

$\nabla\phi=14i-12j-10k$

ii $\nabla\cdot A=\left(\frac{\partial i}{\partial x}+\frac{\partial j}{\partial y}+\frac{\partial k}{\partial z}\right)\cdot a_x i+a_y j+a_z k$

$=\frac{\partial a_x}{\partial x}+\frac{\partial a_y}{\partial y}+\frac{\partial a_z}{\partial z}$

$=\frac{\partial}{\partial x}(x^2y)+\frac{\partial}{\partial y}(xy+yz)+\frac{\partial}{\partial z}(xz^2)$

$=2xy+(x+z)+x2z$

$=2(1)(2)+(1+1)+(1)2(1)$

$=4+2+2$

at $(1,2,1)$

$\nabla\cdot A=4+2+2$

$\nabla\cdot A=8$

(iii) $\nabla\times B=\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3x & 2xy \end{vmatrix}$

$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix}-j\begin{vmatrix} \partial/\partial z & \partial/\partial x \\ yz & 2xy \end{vmatrix}+k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$

$i(\partial/\partial y(2xy)-\partial/\partial z(-3xz)-j(\partial/\partial x(2xy)-\partial/\partial z(yz))]+k[\partial/\partial x(-3xz)-\partial/\partial y(yz)]$

$\nabla\times B=i(2x+3x]-j(2y-y]+k[-3z-\cancel{z}]$ at point $(1,2,1)$

$k(-3(1)-(1)$

iv grad div A

$$\operatorname{div} A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) a_x i + a_y j + a_z k$$

$$= \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$= \frac{\partial (x^2 y)}{\partial x} + \frac{\partial}{\partial y}(xy + y^2) + \frac{\partial}{\partial z} x z^2$$

$$= 2xy + (x+2) + x2z$$

$$\text{grad div} A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot (2xy + (x+z) + x2z)$$

$$= (2y + z + 2z) i + j(2x) + (1+2)k$$

at point (1, 2, 1)

$$\text{grad div} A = (2(2) + (1) + 2(1)) i + (2(1)) j + (1+2) k$$

$$\nabla \nabla \cdot A = 7i + 2j + 3k$$

v curl curl A

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+y^2) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy+y^2) \end{vmatrix}$$

$$i[0 - y] - j[z^2 - 0] + k(y - x^2)$$

$$= -yi - z^2 j + (y - x^2)k$$

$$+ k\left[\partial/\partial x(-z^2) - \partial/\partial y(-y)\right]$$

(y z)]

$$\text{curl curl } A = i(1 + 2z] - j(-2x + 0] + k(0 + 1]$$

$$\text{curl curl } A = (1 + 2z) i + 2xj + k$$

at point (1, 2, 1)

$$\text{curl curl } A = (1 + 2z) i + 2xj + k \text{ at point } (1, 2, 1)$$

$$\text{curl curl } A = (1 + 2(1)) i + 2(1) j + k$$

$$= 3i + 2j + k$$

•) $\left|\frac{d^2 r}{dt^2}\right|$ at $t=0$

$$\frac{d^2 r}{dt^2} = 2i + 18\sin 30(0)j + 12e^{(0)}$$

$t=0$ $2i + 12k$

$$\left|\frac{d^2 r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$$

$$= 12.165 \simeq 12.17$$