$A \times B = 5(i - j) - 4(2k)$

$(1,2,1) = 5(i) - (2j) - 4(1)k$

$= 5i - 2j - 4k$

iv) gradient $\Delta = \Delta \cdot \Delta A$

$= \frac{d}{dx} \cdot \Delta A + \frac{d}{dy} \cdot \Delta A + \frac{d}{dz} \cdot \Delta A$

recall $\Delta A = 2xy + x + z + 2xz$

$\Delta \cdot \Delta A = \frac{d}{dx}(2xy + x + z + 2xz)_i + \frac{d}{dy}(2xy, x + z + 2xz)_j + \frac{d}{dz}(2xy + x$

$\Delta \cdot \Delta A = (2y + 1 + 2z)_i + (2x)_j + (1 + 2x)k$

at point $(1, 2, 1)$

$= 2(2) + 1 + 2(1)_i + 2(1)_j + 1 + 2(1)k$

$= (4 + 1 + 2)_i + 2j + (1 + 2)k$

$\Delta \cdot \Delta A = 7i + 2j + 3k$

$curl\ curl\ A = curl\ (curl\ A)$

$$curl\ A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$$curl\ A \Rightarrow \begin{vmatrix} d/dy & d/dz \\ xy + yz & xz^2 \end{vmatrix} - j \begin{vmatrix} d/dz & d/dz \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} d/dx & d/dy \\ x^2y & (xy + yz) \end{vmatrix}$$

$curl\ A = i\left(\frac{d}{dy} \cdot xz^2 - \frac{d}{dz}(xy - yz)\right) - j\left(\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right) + k\left(\frac{d}{dx}(xy + yz) - \frac{d}{dy}\right.$

$\therefore i(0 - y) + -j(z^2 - 0) + k(y \cdot x^2)$

$curl\ A = -yi - z^2j + (y - x^2)k$

$curl\ A = -yi - z^2j + (y - x^2)k$

) $curl\ curl\ A = \begin{vmatrix} i & j & k \\ d/dx & d/dx & d/dz \\ -y_i & -z^2 & y - x^2 \end{vmatrix}$

$i\left(\frac{d}{dy}(y - x^2) - \frac{d}{dz} - z^2\right) - j\left(\frac{d}{dx} \cdot y - x^2 - \frac{d}{dz} \cdot y\right) + k\left(\frac{d}{dx} \cdot x^2 - \frac{d}{dy} \cdot (y)\right)$

$$\therefore \text{curl curl } A = (1+2z)\,i - j\,(-2x-0) + k\,(0-(-1))$$

$$= (1+2z)\,i + 2x\,j + k$$

$$\text{curl curl } A = (1+2z)\,i + 2x\,j + k$$

$$= (1+2(0))\,i + 2(0)\,j + k$$

$$= 3i + 2j + k$$

$$\text{curl curl } A = 3i + 2j + k$$

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Assignment

1.) (i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpeting the result in physical or in order terms

(ii) using Balance law - law of conservation of mass to forming a differential equation from existing algebraic equation of the system

2.) $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$

$\frac{d^2r}{dt^2}\Big|_{t=0} = 2i + 18\sin 3(0)\,j + 12e^{2(0)}k$

$= 2i + 18\sin 0\,j + 12e^{0}k$

$= 2i + 18\times 0\,j + 12\times 1k$

$= 2i + 12k$

(iii) $\left|\frac{d^2r}{dt^2}\right| = |2i + 12k|$

$= \sqrt{(2)^2 + (2k)^2}$

$= \sqrt{4\times 1\cdot 1 + 144\times k\cdot k}$

$= \sqrt{4\times 1 + 144\times 1} = \sqrt{4+144}$

$= \sqrt{148}$

(iv) $\left|\frac{d^2r}{dt^2}\right| = 2\sqrt{37}$

$\frac{d^2r}{dt^2} = 2\sqrt{37} = 12.165$

$\simeq 12.17$

3) at point (1, 2, 1)

$A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$B = y\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \frac{d\phi}{dx}i + \frac{d\phi}{dy}j + \frac{d\phi}{dz}k$

$\nabla\phi = \frac{d(3x^2y + 3y^2 - 4y^2z^2 - 3)}{dx}i + \frac{d(3x^2y + xyz - 4y^2z^2 - 3)}{dy}j +$

$+ \frac{d(3x^2y + xyz - 4y^2z^2 - 3)}{dz}k$

$\nabla\phi = (6xy + yz)\,i + (3x^2 + xz - 8yz^2)\,j + (xy - 8y^2z)\,k$

at point 1, 2, 1

$\nabla\phi = (6(1)(2) + (2)(1))\,i + (3(1^2) + (1)(1) - 8(2)(1^2))\,j + ((1)(2) - 8(2^2)(1))\,k$

$\nabla\phi = (12 + 2)\,i + (3 + 1 - 16)\,j + 2 - 32\,k$

$\nabla\phi = 4\,i - 12\,j - 30\,k$

ii) $\nabla \cdot A = \frac{d}{dx}A_1 + \frac{d}{dy}\cdot A_j + \frac{d}{dz}\cdot A_z$

$\nabla A = 2xy\,i \cdot i + (x + z)\,j \cdot j + (2xz)\,k \cdot k$

$= 2xy + (x + z)2xz$

$\nabla A(1, 2, 1) = 2(1)(2) + (1) + (1) + 2(1)(1)$

$= 4 + 2 + 2$

$\nabla A = 8$

iii) $A \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= \left(\frac{d}{dy}\cdot 2xy - \frac{d}{dz}\cdot -3xz\right)i - \left(\frac{d}{dx}\cdot 2xy - \frac{d}{dz}\cdot yz\right)j + k\left(\frac{d}{dx}\cdot -3xz - \frac{d}{dy}\cdot yz\right)$

$A \times B = (2x - (-3x))\,i - (2y - y)\,j + (-3z - z)\,k$

$A \times B = 5x\,i - y\,j - 4z\,k$