Question 1

(i) Mathematical modeling can be defined as the process of using mathematical equations and interpreting the result in physical or other terms (simulation).

(ii) Methods include

(a) Forming a differential equation from an existing algebraic equation of the system

(b) Using law of conservation of mass

Question 2

$$\bar{r} = (t^2 + 3t)\hat{i} - 2\sin 3t\,\hat{j} + 3e^{2t}k$$

(i) $$\frac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\,\hat{j} + 6e^{2t}k$$

(ii) $$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) = 2\hat{i} + 18\sin 3t\,\hat{j} + 12e^{2t}k$$

(iii) $$\left|\frac{d^2r}{dt^2}\right| \text{ at } t = 0$$

$$\frac{d^2r}{dt^2} \text{ at } t = 0 = 2\hat{i} + 18\sin 3(0)\hat{j} + 12e^{2(0)}k$$

$$\frac{d^2r}{dt^2} = 2\hat{i} + 12k$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} \qquad \left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2+(12)^2} = \sqrt{4+144} = \sqrt{148} = 2\sqrt{37}$$

Question 3

$A = x^2 y\hat{i} + (xy + yz)\hat{j} + xz^2 k$

$B = yz\hat{i} - 3xz\hat{j} + 2xyk$ are

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$(1,2,1)$

(i) $\nabla\phi$

$$\nabla\phi = \frac{d\phi}{dx} i + \frac{d\phi}{dy} + \frac{d\phi}{dz} k$$

$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2z)k$

at point $(1,2,1)$

$\nabla\phi = (6(1)(2) + (2)(1))\hat{i} + (3(1)^2 + (1)(1) - 8(2)(1)^2)\hat{j} + ((1)(2) - 8(2)^2(1))k$

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30k$

$\nabla\phi = 2(7\hat{i} - 6\hat{j} - 15k)$

(ii) $\nabla \cdot A =$

$$\frac{dd_x}{dx} + \frac{d_y}{d_y} + \frac{dd_z}{dz}$$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$\nabla \cdot A = 8$

(iii) $\nabla \times B$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\nabla \times B = \hat{i} \begin{vmatrix} \frac{d}{dy} & \frac{d}{dz} \\ -3xz & 2xy \end{vmatrix} - \hat{j} \begin{vmatrix} d/dx & d/dz \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ yz & -3xz \end{vmatrix}$$

$$\nabla \times B = \left(\frac{d}{dy}(2xy) - \frac{d}{z}(-3xz)\right) - \hat{j}\left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right)$$
$$+ k\left[\frac{d}{dx}(3xz) - \frac{d}{dy}(yz)\right)$$

$$\nabla \times B = \hat{i}(2x+3x) - \hat{j}(2y-y) + k[-3z-z)$$

$$\nabla \times B = 5x\hat{i} - y\hat{j} - 4zk$$

at pnt $(1,2,1)$

$$\nabla \times B = 5(1)\hat{i} - 2\hat{j} - 4(1)k$$

(1) k $\nabla \times B = \underline{5\hat{i} - 2\hat{j} - 4k}$

(iv) $\text{grad div } A = \nabla \cdot (\nabla A) - \nabla^2 A$

$$\text{grad div} A = \frac{d}{dx}\frac{dA}{dx}\hat{i} + \frac{d}{dy}\frac{dA}{dy} + \frac{d}{dz}\frac{dA}{dz}$$

$$A = x^2y\,\hat{i} + (xy + yz)\hat{j} + xz\,k$$

$$\text{div} A = 2xy + (x+z) + 2xz$$

$$\text{grad div} A = \frac{d}{dx}\frac{dA}{dx}\hat{i} + \frac{d}{dy}\frac{dA}{dy}\hat{j} + \frac{d}{dz}\frac{dA}{dz}k$$

$$\text{grad div} A = (2y + 1 + 2z)\hat{i} + (2x)\hat{j} + (1+2x)k$$

$$\text{grad div} A = (2(2) + 1(2(1))\hat{i} + 2(1)\hat{j} + (1+2(1))k$$

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$$\text{grad div} A = 7\hat{i} + 2\hat{j} + 3k$$

(c) Curl Curl A

$$\text{Curl} A = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix} = \hat{i}\begin{vmatrix} \frac{d}{dy} & \frac{d}{dz} \\ (xy+yz) & xz^2 \end{vmatrix} - \hat{j}\begin{vmatrix} d/dx & \frac{d}{dz} \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ x^2y & (xy+yz) \end{vmatrix}$$

$$\text{Curl} A = \hat{i}\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right] - \hat{j}\left[\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right] + k\left[\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right]$$

$$\text{Curl} A = y\hat{i} - z^2\hat{j} + (y - x^2)k$$

$$\text{Curl Curl} A = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$\text{Curl Curl} A = \hat{i}\left[\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right] - \hat{j}\left[\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right] + k\left[\frac{d}{dx}(-z)^2 - \frac{d}{dy}(-y)\right]$$

$$\text{Curl Curl} A = i(1+2z) - j(-2x+0)] + k[0+1]$$

$$\text{Curl Curl} A = (1+2z)i + 2xj + k$$

$$\text{Curl Curl} A\ (1,2,1) = (1+2(1))\hat{i} + 2(1)\hat{j} + k$$

$$\text{Curl Curl} A = 3\hat{i} + 2\hat{j} + k$$