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COURSE: ENG 282

MATRIC NO: 16/ENG06/058

DEPARTMENT: MECHANICAL ENGINEERING

LEVEL: 200

Question 1

i. A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modelling.

ii. Functional modeling
Architectural modeling[CPC]

Question 2

$r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

(i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\, j + 6e^{2t} k$

(ii) $\frac{d^2r}{dt^2} = (2)i + 18\sin 3t\, j + 12e^{2t} k$

(iii) $\left|\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2 + (12)^2} = 12.17$

Question 3

$A = x^2y\, i + (xy + yz)j + xz^2 k$

$B = yz\, i - 3xz\, j + 2xy\, k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(i) $\nabla\phi = \left(\frac{d}{dx} + \frac{d}{dy} + \frac{d}{dz}\right)(3x^2y + xyz - 4y^2z^2 - 3)$

$= (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

$= (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(2) - 8(2)(1)^2)j + (1\times 2 - 8(2)^2(1)$

$)k$

$\nabla\phi = 14i - 12j - 30k$

(ii) $\nabla \cdot A = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}k\right)(x^2y\, i + (xy + yz)j + xz^2 k)$

$= (2xy\, i + y + z^2)i + (x^2 + (x + z))j + (y + 2xz)k$

$= (2\times 1\times 2 + 2 + 1^2)i + (1^2 + (2))j + (2 + 2(1)(1))k$

$\nabla \cdot A = 6i + 3j + 4k$

Question 2

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$= (2xy\,i + y + z^2)i + (x^2 + (x+z))j + (y+2xz)K$

$= (2\times 1\times 2 + 2 + 1^2)i + (1^2 + (2))j + (2+2(1)(1))K$

$\nabla\cdot A = 6i + 3j + 4K$

(iii) $\nabla \times B = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ -3xz & 2xy \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ yz & 2xy \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ yz & -3xz \end{vmatrix} K$

$= (2x + 3x) i - (2y - y) j + (-3z - z) K$

$= 5x\, i - y\, j - 4z\, K$

$\nabla \times B = 5(1)\, i - (2)\, j - 4x(1) K$

$\nabla \times B = 5i - 2j - 4K$

(iv) grad div A

$\text{div } A = 2xy\, i + (x + z) j + 2xz \cdot K$

$\text{grad div } A = \left(\frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} K \right) (2xy\, i + (x+z) j + 2xz K)$

$= (2y + (z+1) + 2z) i + (2x) j + ((x+1) + 2x) K$

$= (2(2) + (1+1) + 2(1)) i + (2(1)) j + (2 + 2(1)) K$

$\text{grad div } A = 8i + 2j + 4K$

(v) $\text{curl curl } A = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ (xy+yz) & xz^2 \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ x^2 y & xz^2 \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ x^2 y & (xy+yz) \end{vmatrix} K$

$\text{curl } A = (z^2 - y) i - (z^2) j + (y - x^2) K$

$\text{Curl curl } A = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ (z^2 - y) & -z^2 & (y - x^2) \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ -z^2 & (y-x^2) \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ (z^2-y) & (y-x^2) \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ (z^2-y) & -z^2 \end{vmatrix} K$

$= (1 - x^2 + 2z) i - (y - 2x - 2z - y) j + (z^2 - 1) K$

$\text{Curl Curl } A = (i - 1^2 + 2(1)) i - (2 - 2(1) - 2(1) - 2) j + (1^2 - 1) k$

$\text{Curl Curl } A = 2i + 4j$