HALIM NOBERT UDOCHUKWUKA
ENG 282: ENGINEERING MATHEMATICS II
COMPUTER-ENGINEERING | 16/ENG02/021
200 LEVELS
7TH OF APRIL, 2018.

ASSIGNMENT (QUESTIONS & ANSWERS)

1a) Mathematical modelling is the art of translating problems from an application area into traceable mathematical formulations whose theoretical and numerical analysis provides insights, answers and guidance, useful for the originating application.

1b) In chemical engineering => chemical Equilibrium
In Electrical engineering => power supply network optimization

2) $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

$\left.\frac{d^2r}{dt^2}\right|$ at $t=0$

$= 2i + 18\sin 3t\, j + 12e^{2t}k$

$\left|\frac{d^3r}{dt^3}\right|$ at $t=0$

$= 2i + 18\sin(3\times0)j + 12e^{2(0)}k$

$\left|\frac{d^3r}{dt^3}\right| = \sqrt{4+144} = \sqrt{148}$

$= 12.16551$

3) $A = x^2yi + (xy+yz^3)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$= 3x^2y + xy - 4yz^2 z^2 - 3$

i) $\nabla\phi$ at point $(1,2,1)$

$\nabla\phi = \frac{\delta\phi}{\delta x}i + \frac{\delta\phi}{\delta y}j + \frac{\delta\phi}{\delta z}k$

$\frac{\delta\phi}{\delta x} = 6xy + y_2$

$\frac{\delta\phi}{\delta y} = 3x^2 + xz - 8yz^2$

$\frac{\delta\phi}{\delta z} = xy - 8y^2z$

$\nabla\phi = (6xy+yz)i + (3x^2+xz-8yz^2)j + (xy-8y^2z)k$ at point $(1,2,1)$

$\nabla\phi = (12+2)i + (3+1-8(2)(2))j + (2-32)k$

$= 14i + (4-16)j + (-30)k$

$= 14i - 12j - 30k$

$\nabla\phi = 14i - 1$

a) ∇A

$\nabla A = \frac{\delta}{\delta x} i$

$\nabla A = \frac{\delta}{\delta x}$

$\nabla A = 2$

$= 2$

$= 2$

$\nabla A =$

iii) $\nabla\times$

$i \left[\right.$

$\nabla\phi = 14i - 12j - 304$

i) ∇A

$\nabla A = \frac{\delta}{\delta x} i + \frac{\delta}{\delta y} j + \frac{\delta}{\delta z} k \cdot (x^2 y i + (xy+yz) j + xz^2 k)$

$\nabla A = \frac{\delta}{\delta x}(x^2 y) + \frac{\delta}{\delta y}(xy+yz) + \frac{\delta}{\delta z}(xz^2)$

$\nabla A = 2xy + (x+z) + 0 + (2xz)$

$= 2(1)(2) + (1+1) + (2 \times 1 \times 1)$

$= 4 + 2 + 2$

$\nabla A = 8$

ii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ yz & -3xz & 2xy \end{vmatrix}$

$i\left[\frac{\delta}{\delta y}(2xy) + \frac{\delta}{\delta z}(3xz)\right] - j\left(\frac{\delta}{\delta x}(2xy) - \frac{\delta}{\delta z}(yz)\right)$

$k\left[\frac{\delta}{\delta z}(-3xz) - \frac{\delta}{\delta y}(yz)\right]$

$i(2x+3x) - j(2y-y) + k(-3z-z)$

$i(5x) - j(y) + k(-4z)$

$5xi - yj - 4zk$

at point $(1, 2, 1)$

$\nabla \times B = 5i - 2j - 4k$

ii) grad div A $\overrightarrow{\nabla A}$

$\nabla \cdot A = 2xy + (x+z) + 2zx$

$\nabla(\nabla A) = \frac{d(\nabla A)}{dx} i + \frac{d(\nabla A)}{dy} j + \frac{d(\nabla A)}{dz} k$

$\nabla(\nabla A) = i(2y + 1 + 2z) + 2xj + (1+2x)k$

$= i(4+1+2) + 2j + 3k$

$= 7i + 2j + 3k$

$\nabla \times (\nabla \times A)$ $\vec{A} = x^2 y i + (xy + yz) j + xz^2 k$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2 y & (xy + yz) & (xz^2) \end{vmatrix}$$

$i(0-y) - j(z^2 - 0) + k(x^2 + (y)) - yi - j(z^2) + k(x - y)$

$\nabla \times \vec{A} = -yi - j(z^2) + k(x^2 - y)$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} \frac{d}{dx} i & \frac{d}{dy} j & \frac{d}{dz} k \\ -y & -z^2 & x^2 y \end{vmatrix}$$

$= i(1 + 2z) - j(0 - 2x) + k(1 - 0)$

$= (1+2z)i + 2xj + k$

$= (1 + 2(1))i + 2j + k$

$\nabla \times (\nabla \times \vec{A}) = 3i + 2j + k$