Uchechukwu Miracle N.K
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ENG 282

(1)
(a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the results in physical and other terms

(b) (i) Exponential growth/decay (use of ODE)
(ii) Mixing problems

(2) $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$

(i) $\frac{\partial r}{\partial t} = (2t - 3)i - 6\cos 3t j + 6e^{2t} k$

(ii) $\frac{\partial^2 r}{\partial t} = 2i + 18\sin 3t j + 12e^{2t} k$

(iii) $\left.\frac{\partial^2 r}{\partial t^2}\right|_{t=0} = 2i + 12k$

$\left|\frac{\partial^2 r}{\partial t^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{4+144} = \sqrt{148}$

$= 2\sqrt{37} = 12.7$

(3) $A = x^2 y i + (xy + y^2) j + xz^2 k$

$B = yzi - 3xz + 2xyk$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

(i) $\nabla\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z}$

$\frac{\partial \phi}{\partial x} = 6xy + y^2$

$\partial\phi/\partial z = xy - 8y^2 z$

$\partial\phi/\partial z = 3x^2 + xz - 8yz^2$

At $(1, 2, 1)$

$\partial\phi/\partial x = 12 + 2 = 14$

$\partial\phi/\partial y = 3 + 1 - 16 = -12$

$\partial\phi/\partial z = 2 - 32 = -30$

$\nabla\phi = 14i - 12j - 30k$

(ii) $\bar{\nabla} \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x i + a_y j + a_z k$

$\bar{\nabla} \cdot A = 2xy + (x+z) + 2xz$

At (1, 1, 1)

$\bar{\nabla} \cdot A = 4 + 2 + 2 = 8$

(iii) $\bar{\nabla} \cdot B$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x+3x) - j(2y-y) + k(-3z-z)$

$= 5xi - yj - 4zk$

At (1,2,1)

$\bar{\nabla} \times B = 5i - 2j - 4k$

(iv) grad div A

grad $(2xy + (x+z) + 2xz)$

let div $A = C = \bar{\nabla} A$

$\bar{\nabla}(\bar{\nabla} A) = \bar{\nabla} C = i\frac{\partial c}{\partial x} + j\frac{\partial c}{\partial y} + k\frac{\partial c}{\partial z}$

$= i(2y+1+2z) + j(2x) + k(1+2x)$

At (1,2,1)

$\bar{\nabla} \cdot C = i(4+1+2) + j(2) + k(1+2)$

$= 7i + 2j + 3k$

(v) Curl curl A

Curl $A = \bar{\nabla} \times A$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy+yz) & xz^2 \end{vmatrix}$$

$= i(0-y) - j(z^2-0) + k(y-x^2)$

$= -yi - z^2 j + k(y-x^2)$

At (1,2,1)

Curl $A = -2i - j + k$

Curl Curl $A = \bar{\nabla} \times (\bar{\nabla} \times A)$

$\bar{\nabla} \times (\bar{\nabla} \times A)$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$= i\,(1+2z) - j\,(-2x - 0) + k\,(0+1)$

$= i\,(1+2z) + 2x^2 j + k$

At point $(1, 2, 1)$

$\nabla^2 \times (\bar{\nabla} \times A) = i(1+\cancel{2}(1)) + 2(1)^2 j + k)$

$= 3i + 2j + k$