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16/ENGG06/1001

Mech Engineering

1.) Mechanical Modelling.

This is the process of setting up a model, solving the mathematically, and interpreting the result in physical or in order items.

ii) Using balance law \rightarrow law of Conservation of mass. Forming a differential equation from an existing algebraic equation of the system.

$$2.) \quad r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$$
$$\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t} k$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt} \left(\frac{dr}{dt} \right)$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k.$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3(0) j + 12e^{2(0)} k.$$

$$2i + 18\sin 0 j + 12e^0 k$$

$$2i + 18 \times 0 j + 12 \times 1 k$$

$$= 2i + 12k$$

$$iv) \quad \left| \frac{d^2r}{dt^2} \right| = |2i + 12k|$$

$$\frac{2(3x^2y + xyz - 4y^2z^2 - 3)}{2z} \quad \frac{dy}{dz}$$

$$\vec{\nabla}\phi = (6xy + yz)\mathbf{i} + (3x^2 + xz - 8yz^2)\mathbf{j} + (xy - 8y^2z)\mathbf{k}$$

at point (1, 2, 1)

$$\vec{\nabla}\phi = (6(1)(2) + (1)(2))\mathbf{i} + (3(1)^2 + (1)(1) - 8(2)(1)^2)\mathbf{j} + ((1)(2) - 8(2^2)(1))\mathbf{k}$$

$$\vec{\nabla}\phi = (14\mathbf{i} - 12\mathbf{j} - 30\mathbf{k})$$

(1, 2, 1)

$$\nabla \times B = 5xz \mathbf{i} - yz \mathbf{j} - 4z \mathbf{k}$$

$$\begin{aligned} |\nabla \times B| &= 5(1) \mathbf{i} - (2) \mathbf{j} - 4(1) \mathbf{k} \\ &= 5\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \end{aligned}$$

$$\begin{aligned} \text{v) grad. div } A &= \nabla \cdot \nabla A \\ &= \frac{\partial}{\partial x} \cdot \nabla A \mathbf{i} + \frac{\partial}{\partial y} \cdot \nabla A \mathbf{j} + \frac{\partial}{\partial z} \cdot \nabla A \mathbf{k} \end{aligned}$$

recall $\nabla A = 2xy \mathbf{i} + x \mathbf{j} + z \mathbf{k} + 2xz \mathbf{k}$

$$\nabla \cdot \nabla A = \frac{\partial}{\partial x} (2xy + x + z + 2xz) \cdot \mathbf{i} + \frac{\partial}{\partial y} (2xy + x + z + 2xz) \cdot \mathbf{j} + \frac{\partial}{\partial z} (2xy + x + z + 2xz) \cdot \mathbf{k}$$

$$= 2y \mathbf{i} + (1 + 2z) \mathbf{j} + (1 + 2x) \mathbf{k}$$

$$\nabla \cdot \nabla A = (2y + 1 + 2z) \mathbf{i} + (2x) \mathbf{j} + (1 + 2z) \mathbf{k}$$

at point (1, 2, 1)

$$= 2(2) + 1 + 2(1) \mathbf{i} + 2(1) \mathbf{j} + (1 + 2(1)) \mathbf{k} = (4 + 1 + 2) \mathbf{i} + 2 \mathbf{j} + (1 + 2) \mathbf{k}$$

$$\nabla \cdot \nabla A = 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

$$\text{grad div } A = 7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

$$\text{Curl } A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & (xy + yz) & xz^2 \end{vmatrix}$$

$$\text{Curl } A = \mathbf{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (xy + yz) & xz^2 \end{vmatrix} - \mathbf{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ 2xy & xz^2 \end{vmatrix} + \mathbf{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 2xy & (xy + yz) \end{vmatrix}$$

Curl A

$$i(0-y) + (-)(z^2-0) + k(y-z^2)$$

$$\text{Curl } A = -yi - z^2j + (y-z^2)k$$

$$\text{curl } A = -yi - z^2j + (y-z^2)k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & y-z^2 \end{vmatrix}$$

$$i \left(\frac{\partial}{\partial y} \cdot (y-z^2) - \frac{\partial}{\partial z} \cdot (-z^2) \right) - j \left(\frac{\partial}{\partial x} \cdot (y-z^2) - \frac{\partial}{\partial z} \cdot (-y) \right)$$

$$+ k \left(\frac{\partial}{\partial x} \cdot (-z^2) - \frac{\partial}{\partial y} \cdot (-y) \right)$$

$$\text{Curl Curl } A = (1+z^2)i - (-)(-2z-0) + k(0-(-1))$$

$$\text{Curl Curl } A = (1+z^2)i + 2zj + k$$

$$\text{Curl Curl } A = (1+z^2)i + 2zj + k$$

$$= (1+z^2)i + 2zj + k$$

$$= 3j + 2j + k = 3i + 2j + k$$

$$\text{curl curl } A = 3i + 2j + k$$