QUESTION 1

(i) Define ~~Modelling~~. Mathematical Modelling

A Mathematical Model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed Mathematical modelling.

It is therefore an art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insights answers and guidance useful for the originating application.

(ii) Two ways of obtaining ~~modelling~~ for engineering systems.

- using the balance law.

- Forming a differential equation from an existing equation of the system.

Curl ~~curl~~ A:

$$(\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & xy+yz & xz^2 \end{vmatrix}$$

$$= i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy+yz & xz^2 \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & xy+yz \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right]$$
$$+ k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$= i[0-y] - j[z^2-0] + k[y-x^2]$$

$$(\nabla \times A) = -yi - z^2j + (y-x^2)k$$

$\nabla \times (\nabla \times A)$:

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & y-x^2 \end{vmatrix}$$

$$= i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z^2 & y-x^2 \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ -y & y-x^2 \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -y & -z^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right]$$
$$+ k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$$

$$= i[1+2z] - j[-2x-0] + k[0+1]$$

$$\nabla \times (\nabla \times A) = [1+2z]i + 2xj + k$$

at (1,2,1):

$$\nabla \times (\nabla \times A) = [1+2(1)]i + 2(1)j + k$$
$$= (1+2)i + 2j + k$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k$$

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(ii) $\nabla\cdot A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$\nabla\cdot A = (2xy) + (x + z)\,j + 2xz\,k$

at 1,2,1 :

$\nabla\cdot A = [2(1)(2)] + [1+1] + [2(1)(1)]$

$= 4 + 2 + 2$

$\nabla\cdot A = 8$

(iii) $\nabla \times B$:

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\nabla\times B = i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ yz & -3xz \end{vmatrix}$$

$= i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xy)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}yz\right] + k\left[\frac{\partial}{\partial x}(-3xy) - \frac{\partial}{\partial y}yz\right]$

$= i\,(2x + 0) - j\,(2y - y) + k\,(-3y - z)$

$= 2x\,i - (2y - y)\,j - (3y + z)\,k$

$= 2x\,i - (2y - y)\,j - (3y + z)\,k$

$\nabla\times B = (2x)\,i - (2y - y)\,j - (3y + z)\,k$

(iv) ~~grad div A:~~

~~div A =~~

$\nabla\times B$ at $(1,2,1)$ $= [2(1)]\,i - [2(2) - 2]\,j - [3(2) + 1]\,k$

$\nabla\times B \Rightarrow 2i - [4 - 2]\,j - [6 + 1]\,k$

$\nabla\times B = 2i - 2j - 7k$

(v) grad div A:

$\nabla\cdot A = (2xy) + (x + z) + (2xz)$

$\nabla(\nabla\cdot A) = (2y + 2z)\,i + (2x)\,j + (1 + 2x)\,k$

at $(1,2,1)$:

$\nabla(\nabla\cdot A) = [2(2) + 2(1)]\,i + [2(1)]\,j + (1 + 2(1))\,k$

$= (4 + 2)\,i + 2j + (1 + 2)\,k$

$\nabla(\nabla\cdot A) = 6i + 2j + 3k$

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QUESTION 2:

$r = (t^2+3t)i - 2\sin 3tj + 3e^{2t}K.$

i. $\frac{dr}{dt} \Rightarrow \frac{d\left((t^2+3t)i - 2\sin 3tj + 3e^{2t}K\right)}{dt}$

$\frac{dr}{dt} \Rightarrow (2t+3)i - 6\cos 3tj + 6e^{2t}K$

ii) $\frac{d^2r}{dt^2} = \frac{d\left((2t+3)i - 6\cos 3tj + 6e^{2t}K\right)}{dt}$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3tj + 12e^{2t}K$

iii) At $t = 0$:

$\frac{d^2r}{dt^2} = 2i + 18\sin 3(0)j + 12e^{2(0)}K$

$= 2i + 0j + 12K$

$\frac{d^2r}{dt^2} = 2i + 12K$

QUESTION 3:

$A = x^2yi + (xy + yz)j + xz^2K$

$B = yzi - 3xzj + 2xyK.$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3,$

At the point; $(1, 2, 1)$:

i.) $\nabla\phi$:

$\Rightarrow (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)K$

i) $\nabla\phi = [6(1)(2) + 2(1)]i + [3(1)^2 + 1(1) - 8(2)(1)^2]j + [1(2) - 8(2)^2(1)]K$

$= (12 + 2)i + (6 + 1 - 10)j + (2 - 32)K$

i) $\nabla\phi = 14i - 3j - 30K$