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MATRIC NO 16/ENG07/021

Answers

1. Mathematical modelling is the art of translating problems from an application area into tracable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

1b. In chemical engineering ⇒ chemical equilibrum
In Electrical engineering ⇒ Power supply network optimization

2. $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$

$$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$$= 2i + 18\sin 3t\,j + 12e^{2t}k$$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$$= 2i + 18\sin(3\times 0)j + 13e^{2(0)}k$$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4+144} = \sqrt{148} \approx 12.1655$$

3. $A = x^2yi + (xy+yz)j + xz^2k$
$B = yzi - 3xzj + 2xyk$
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi$ at point $(1,3,1)$

$$\nabla\phi = \frac{\delta\phi}{\delta x}i + \frac{\delta\phi}{\delta y}j + \frac{\delta\phi}{\delta z}k$$

$$\frac{\delta\phi}{\delta x} = 6xy + yz$$

$$\frac{\delta\phi}{\delta y} = 3x^2 + xz - 8yz^2$$

$\frac{\partial \phi}{\partial z} = xy - 8y^2 z$

$\nabla \phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

at point $(1,2,1)$

$\nabla \phi = (12+2)i + (3+1-8(2)(2))j + (2-32)k$

$= 14i + (4-16)j + (-30)k$

$= 14i - 12j - 30k$

$\nabla \phi = 14i - 12j - 30k$

ii) ∇A

$\nabla A = \frac{\delta}{\delta x} i + \frac{\delta}{\delta y} j + \frac{\delta}{\delta z} k \cdot (x^2 y i + (6xy + yz)j + xz^2 k)$

$\nabla A = \frac{\delta}{\delta x}(x^2y) + \frac{\delta}{\delta y}(6xy + yz) + \frac{\partial}{\partial z}(xz^2)$

$\nabla A = 2xy + 6x + z + 0 + (2xz)$

$= 2(1)(2) + (1+1) + (2 \times 1 \times 1)$

$= 4 + 2 + 2$

$\nabla A = 8$

iii. $\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ yz & -3xz & 2xy \end{vmatrix}$

$i\left(\frac{d}{dy}(2xy) + \frac{d(3xz)}{d}\right) - j\left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right)$

$k\left(\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right)$

$i(2x + 3x) - j(2y - y) + k(-3z - z)$

$i(5x) - j(y) + k(-4z)$

$5xi - yj - 4zk$

at point $(1,2,1)$

$\nabla \times B = 5i - 2j - 4k$

iv grad div A $\nabla \cdot \vec{A}$

$\text{Div } A = 2xy + (x + z) + 2zx$

$\nabla(\nabla \cdot A) = \frac{d(\nabla A)}{dx} i + \frac{d(\nabla A)}{dy} j + \frac{d(\nabla A)}{dz} k$

$\nabla(\nabla A) = i(2y+1+2z) + 2xj + (1+2x)k$

$= i(4+1+2) + 2j + 3k$

$= 7i + 2j + 3k$

$\nabla \times (\nabla \times \vec{A})$ $\quad \vec{A} = x^2 y i + (xy + yz)j + xz^2 k$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2 y & (xy+yz) & (xz^2) \end{vmatrix}$$

$i(0-y) - j(z^2 - 0) + k(x^2 - (y)) - yi - j(z^2) + k(x^2 - y)$

$\nabla \times \vec{A} = -yi - j(z^2) + k(x^2 - y)$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & x^2 - y \end{vmatrix}$$

$= i(1 + 2z) - j(0 - 2x) + k(1 - 0)$

$= (1 + 2z)i + 2xj + k$

$= (1 + 2(1))i + 2j + k$

$\nabla \times (\nabla \times \vec{A}) = 3i + 2j + k$