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① Mathematical modelling is a process of setting up a model, solving it mathematically and interpreting the result in physical or other terms.

(a) Numeric method by Euler
(b) Extended method by reduction to Seperable form

② $r = (t^2 + 3t)i - 2\sin 3t\,j + 3e^{2t}k$

$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

③ $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$

④ $\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 12k$

$= \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 12.17$

(3) $A = x^2 y\,i + (xy + yz)\,j + xz^2\,k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

points $(1, 2, 1)$ $\quad x=1,\ y=2,\ z=1$

(i) $\nabla\phi \left[i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz} \right] \phi$

$= (6xy + yz)\,i + (3x^2 + yz - 8yz^2)\,j + (xy - 8y^2 z)\,k$

at point $(1, 2, 1)$

$\nabla\phi = 6(1)(2) + (2)(1)\,i + (3(1)^2 + (2)(1) - 8(2)(1)^2)\,j + ((1)(2) - 8(2)^2(1))\,k$

$\nabla\phi = 14i - 12j - 30k$

(ii) $\nabla \cdot A = \frac{d}{dx}(3x) + \frac{d}{dy}(xy) + \frac{d}{dz}(xz)$

$= 2xy + (x + z) + 2xz$

at points $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2$

$\nabla \cdot A = 8$

(iii) $\nabla \times B = \begin{vmatrix} + & - & + \\ i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\left(\frac{d}{dy}(2xy) - \frac{d}{dz}(-3xz)\right) - j\left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right) + k\left(\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right)$

at points $(1, 2, 1)$

$\nabla \times B = (2(1) + 3(1))\,i - (2(2) - 2)\,j + (-3(1) - 1)\,k$

$= 5i - 2j - 4k$

(iv) grad div A

div A, $\nabla \cdot A = 2xy + (x+z) + (x+z) + 2xz$

$\therefore$ grad div $A = \left(i \frac{d\nabla \cdot A}{dx} + j \frac{d\nabla \cdot A}{dy} + k \frac{d}{dz} \right) \cdot \nabla \cdot A$

$= i \frac{d\nabla \cdot A}{dx} + j \frac{d\nabla \cdot A}{dy} + k \frac{d\nabla \cdot A}{dz}$

$= (2y + z + 2z)\, i + 2x\, j + (1 + 2x)\, k$

at points $(1, 2, 1)$

grad div $A = (2(2) + 1 + 2(1))\, i + 2(1)\, j + (1 + 2(1))\, k$

grad div $A = 7i + 2j + 3k$

(v) Curl Curl A =

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$i\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy + yz)\right] - j\left(\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2 y)\right) + k\left(\frac{d}{dx}(xy + yz) - \frac{d}{dy}(x^2 y)\right)$$

$$\text{Curl } A = -y\, i - z^2\, j + (y - x^2)\, k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$= i\,((1-0) + 2z) - j\,((0 - 2x) + 0) + k\,(0 + 1)$

$= (1 + 2z)\, i + 2x\, j + k$

at point $(1, 2, 1)$

Curl Curl $A = (1 + 2(1))\, i + 2(1)\, j + k$

$= 3i + 2j + k$