1) Mathematical modelling is the process of representing the relationship between two or more quantities, a variable with mathematical equations.

ii) i) By the use of the mass balance equation
ii) By differentiating algebraic equation with constants

Question 2.

$r = (t^2+3t)i - (2\sin 3t)j + 3e^{2t}k$

i) $\frac{dr}{dt} = 2t+3i - 6\cos 3tj + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3tj + 12e^{2t}k$

at $t=0 \Rightarrow 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = 2\sqrt{37}$

Question 3

$A = x^2y\,i + (xy + y_3)\,j + xz^2\,k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(i) $\nabla\phi$

$\Rightarrow \left(\frac{di}{dx} + \frac{dj}{dy} + \frac{dk}{dz}\right)\left[3x^2y + xy_3 - 4y^2z^2 - 3\right]$

$\Rightarrow \frac{d(\phi)}{dx}\,i + \frac{d(\phi)}{dy}\,j + \frac{d\phi}{dz}\,k$

$\Rightarrow (6xy + yz)\,i + (3x^2 + x_3 - 8y_3)\,j + (xy - 8y^2)\,k$

at $(1,2,1) \Rightarrow 14\,i - 12\,j - 30\,k$

(ii) $\nabla \cdot A$

$\Rightarrow \left(\frac{di}{dx} + \frac{dj}{dy} + \frac{dk}{dz}\right) \cdot \left(x^2y\,i + (xy + y_3)\,j + xz^2\,k\right)$

$\Rightarrow \frac{d(x^2y)}{dx} + \frac{d(xy + y_3)}{dy} + \frac{d(xz^2)}{dz}$

$\Rightarrow 2xy + x + z + 2x_3$

at $(1,2,1)$

$2(1)(2) + 1 + 1 + 2(1)(1)$

$4 + 1 + 1 + 2$

$\Rightarrow 8.$

iii) $\nabla \times B$

$$\Rightarrow \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\Rightarrow \left(\frac{\partial}{\partial y}\cdot 2xy - \frac{\partial}{\partial z}\cdot -3xz\right)i - \left(\frac{\partial}{\partial x}\cdot 2xy - \frac{\partial}{\partial z}\cdot(yz)\right)j + \left(\frac{\partial}{\partial x}\cdot -3xz - \frac{\partial}{\partial y}\cdot(yz)\right)k$$

$$\Rightarrow (2x + 3x)i - (2y - y)j + (-3z - z)k$$

$$5xi - yj - 4zk \quad \text{at } (1,2,1)$$

$$5i - 2j - 4k$$

iv) grad div A

$\nabla(\nabla \cdot A)$

$$\Rightarrow \nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot\left(x^2y\,i + (xy + yz)j + xz^2k\right)$$

$$\Rightarrow 2xy + x + z + 2xz$$

$$\nabla(\nabla \cdot A) = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\{2xy + x + z + 2xz\}$$

$$\Rightarrow \frac{\partial}{\partial x}i(2xy + x + z + 2xy) + \frac{\partial}{\partial y}j(2xy + x + z + 2xz) + \frac{\partial}{\partial z}k(2xy + x + z + 2xz)$$

$$= (2y + 2z + 1)i + (2x)j + (2x + 1)k. \quad \text{at } (1,2,1)$$

$$\Rightarrow 7i + 2j + 3k$$

v) Curl (curl A)

$$\Rightarrow \nabla \times (\nabla \times A)$$

$$\Rightarrow (\nabla \cdot A)\nabla - (\nabla \cdot \nabla)A.$$

$$\therefore \nabla \cdot A \Rightarrow 2xy + x + z + 2xz$$

$$\nabla \cdot \nabla \Rightarrow \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)$$

$$\Rightarrow \nabla^2 \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$\Rightarrow \nabla \times (\nabla \times A)$

$= \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)\{2xy + x + z + 2xz\} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) A$

$\Rightarrow (2y + 2z + 1)i + 2xj + (2x + 1)k - (2yi + 2xk)$

$\Rightarrow (2z + 1)i + 2xj + k$ at $(1, 2, 1)$

$\Rightarrow (2(1) + 1)i + 2(1)j + k$

$3i + 2j + k$