

$$(1) f(x) = e^{-0.5x} (4-x) - 2$$

$$f'(x) = -0.5e^{-0.5x} (4-x) - e^{-0.5x}$$

$$\text{Recall: } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \left[\frac{e^{-0.5x} (4-x) - 2}{-0.5e^{-0.5x} (4-x) - e^{-0.5x}} \right]$$

$$\text{for } i=0, x=0.5$$

$$x_{i+1} = 0.5 - \left[\frac{e^{-0.5(0.5)} (4-0.5) - 2}{-0.5e^{-0.5(0.5)} (4-0.5) - e^{-0.5(0.5)}} \right]$$

$$= 0.8389$$

$$\text{for } i=1, x=0.8389$$

$$x_{i+1} = 0.8389 - \left[\frac{e^{-0.5(0.8389)} (4-0.8389) - 2}{-0.5e^{-0.5(0.8389)} (4-0.8389) - e^{-0.5(0.8389)}} \right]$$

$$= 0.8850$$

$$\text{for } i=2, x=0.8850$$

$$x_{i+1} = 0.8850 - \left[\frac{e^{-0.5(0.8850)} (4-0.8850) - 2}{-0.5e^{-0.5(0.8850)} (4-0.8850) - e^{-0.5(0.8850)}} \right]$$

$$= 0.8857$$

$$\text{for } i=3, x=0.8857$$

$$x_{i+1} = 0.8857 - \left[\frac{e^{-0.5(0.8857)} (4-0.8857) - 2}{-0.5e^{-0.5(0.8857)} (4-0.8857) - e^{-0.5(0.8857)}} \right]$$

$$= 0.8857$$

$$\text{for } i=4, x=0.8857$$

$$\text{for } i=5, x=0.8857$$

$$\% \text{ error tolerance} = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

$$\text{for } i=0, x=0.5, \% \text{ error} = 100\%$$

$$\text{for } i=1, x=0.8389$$

$$\% \text{ error}_{\text{tolerance}} = \left| \frac{0.8389 - 0.5}{0.8389} \right| \times 100$$

$$= 40.39\%$$

for $i=2$, $x = 0.8850$

$$\% \text{ error tolerance} = \left| \frac{0.8850 - 0.8889}{0.8850} \right| \times 100$$
$$= 5.28\%$$

for $i=3$, $x = 0.8857$

$$\% \text{ error tolerance} = \left| \frac{0.8857 - 0.8850}{0.8857} \right| \times 100$$
$$= 0.079\%$$

for $i=4$, $x = 0.8857$

$$\% \text{ error tolerance} = \left| \frac{0.8857 - 0.8857}{0.8857} \right| \times 100$$
$$= 0\%$$

for $i=5$, $x = 0.8857$

$$\% \text{ error tolerance} = \left| \frac{0.8857 - 0.8857}{0.8857} \right| \times 100$$
$$= 0\%$$

i	x_i	% Error
0	0.5	100
1	0.8389	40.89
2	0.8850	5.28
3	0.8857	0.079
4	0.8857	0
5	0.8857	0

$$(2) \quad F_D = \frac{0.8V^2}{500 + (\ln V)^3} - 0.02V$$

$$F_D = mg$$

$$m = 3.5 \text{ kg}, \quad g = 9.8 \text{ m/s}^2$$

$$F_D = 9.8 \times 3.5 = 34.3$$

Substitute

$$34.3 = \frac{0.8V^2}{500 + (\ln V)^3} - 0.02V$$

$$0 = \frac{0.8V^2}{500 + (\ln V)^3} - 0.02V - 34.3$$

$$F_D = \frac{0.8V^2}{500 + (\ln V)^3} - 0.02V - 34.3$$

$$F'(V) = ?$$

$$\frac{0.8V^2}{500 + (\ln V)^3}$$

$$\text{from quotient rule: } \frac{U'V - V'U}{V^2}$$

$$U = 0.8V^2$$

$$U' = 0.6V$$

$$V = 500 + (\ln V)^3$$

$$V' = \frac{3(\ln V)^2}{V}$$

$$= \frac{0.6V(500 + (\ln V)^3) - 0.8V^2 \left(\frac{3(\ln V)^2}{V} \right)}{(500 + (\ln V)^3)^2}$$

$$= \frac{300V + (\ln V)^3 \cdot 0.6V - 0.9V(\ln V)^2}{500^2 + \ln V^6}$$

$$F'(V) = \frac{300V + \ln V^3(0.6V) - 0.9V(\ln V)^2}{500^2 + \ln V^6} - 0.02$$

$$x_{i+1} = x_i - \frac{F(x_i)}{F'(x_i)}$$

$$= x_i - \frac{0.8V^2}{500 + (\ln V)^3} - 0.02V - 34.3$$

$$\frac{300V + \ln V^3(0.6V) - 0.9V(\ln V)^2 - 0.02}{500^2 + \ln V^6}$$