$$= i\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy + yz)\right] - j\left[\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right]$$

$$+ k\left[\frac{d}{dx}(xy + yz) - \frac{d}{dy}(x^2y)\right]$$

$$= i[-y] - j(z^2) + k[y - x^2]$$

$$= -yi - z^2j + (y - x^2)k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & y - x^2 \end{vmatrix}$$

$$= i\left[\frac{d}{dy}(y - x^2) - \frac{d}{dz}(-z^2)\right] - j\left[\frac{d}{dx}(y - x^2) - \frac{d}{dz}(-y)\right]$$

$$+ k\left[\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right]$$

$$= 1[1 + 2z] - j[-2x] + k(1)$$

$$= i[1 + 2z] + 2xj + k$$

$$\text{At } (1, 2, 1) = i[1 + 2(1)] + 2(1)j + k$$

$$= 3i + 2j + k$$

At $(1,2,1) = (6\times1\times2+2\times1)i + (3(1)^2+1\times1-8(2)(1))j + (1+2-8(2)^2(1))k$

$= 14i - 12j - 30k$

ii) $\nabla \cdot A = 2xy + (x+z) + 2xz$

At $(1,2,1) = (2\times1+2) + (1+1) + (2\times1\times1)$

$= 4+2+2$

$= 8$

iii) $\nabla \times A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\left[\frac{d}{dy}(2xy) - \frac{d}{dz}(-3xz)\right] - j\left[\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right]$

$+ k\left[\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right]$

$= i[2x+3x] - j[2y-y] + k[-3z-z]$

$= 5xi - yj - 4zk$

At $(1,2,1) = 5(1)i - 2j - 4(1)k$

$= 5i - 2j - 4k$

i) grad div A

$\text{div } A = 2xy + (x+y) + 2xz$

$\text{grad div } A = (2y+1+2z)i + (2x+1)j + (2x)k$

At $(1,2,1) = (2(2)+1+2(1))i + (2(1)+1)j + (2(1))k$

$= 7i + 3j + 2k$

ii) Curl A

$\nabla \times A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy+y^2) & yz^2 \end{vmatrix}$

SULEIMAN ABDULLAHI EZHIN
17/6/GO5/046
MECHATRONICS

1.) Mathematical modelling is the process of translating problems from an application area into mathematical formulations whose theoretical and numerical analysis provide insight, answers and guidance useful for the originating application

ii) A) Using balance law - law of Conservation of mass
b) forming differential equation from an existing algebraic equation of the system

2) $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k$

iii) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0 = 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 18\sin 0\, j + 12e^{0}k$

$= 2i + 0j + 12k$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$ $\sqrt{2^2+0^2+12^2}$

$= \sqrt{148}$

$= 12.17$

(3) $A = x^2yi + (xy+yz)j + xz^3k$

$B = yzi + 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$P(1,2,1)$

i) $\nabla\phi = (6xy+yz)i + (3x^2+xz-8yz^2)j + (xy-8y^2z)k$