SAM-OGBONNA CHIDIEBERE
16/ENG01/018
CHEMICAL ENGINEERING
ASSIGNMENT 3

① a Modelling can be defined as the process of setting up a model solving it mathematically, and interpreting the result in physical or other terms.

b Methods of creating model.
(i) Radio-activity exponential growth/decay.
(ii) Mixing problems.

2 $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$.

(i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t} k$.

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k$

(iii) $\frac{d^2r}{dt^2}$ at $t=0$.

$= 2i + 18\sin 0 j + 12e^{2\times 0} k$.

$\frac{d^2r}{dt^2} = 2i + 12k$.

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2}$

$\left|\frac{d^2r}{dt^2}\right| = 12.17$.

③ $A = xyi + (xy + yz)j + xzk$

$B = yzi - 3xzj + 2xyk$.

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$.

at point(1, 2, 1).

(i) $\nabla\phi = \frac{\delta\phi}{\delta x} i + \frac{\delta\phi}{\delta y} j + \frac{\delta\phi}{\delta z} k$

$$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2 z)\hat{k}$$

at point $(1,2,1)$

$$\nabla\phi = (12+2)\hat{i} + (3+1-16)\hat{j} + (2-32)\hat{k}$$

$$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$$

(ii) $\nabla\cdot A = \frac{\delta a_x}{\delta x} + \frac{\delta a_y}{\delta z} + \frac{\delta a_z}{\delta z}$

$$\nabla\cdot A = 2xy + (x+z) + 2xz.$$

at point $(1,2,1)$.

$$\nabla\cdot A = 4+2+2.$$

$$\nabla\cdot A = 8.$$

(iii) $\nabla\times B = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ b_x & b_y & b_z \end{vmatrix}$

$$\nabla\times B = i\left(\frac{\delta b_z}{\delta y} - \frac{\delta b_y}{\delta z}\right) - j\left(\frac{\delta b_z}{\delta x} - \frac{\delta b_x}{\delta z}\right) + \hat{k}\left(\frac{\delta b_y}{\delta x} - \frac{\delta b_x}{\delta y}\right)$$

$\frac{\delta b_x}{\delta y} = 2x,\ \frac{\delta b_y}{\delta x} = -3x,\ \frac{\delta b_x}{\delta x} = 2y.$

$\frac{\delta b_x}{\delta z} = y,\ \frac{\delta b_y}{\delta x} = -3z,\ \frac{\delta b_x}{\delta y} = z.$

$$\nabla\times B = (2x-(-3x))\hat{i} - (2y-y)\hat{j} + (-3z-2)\hat{k}$$

$$\nabla\times B = 5x\hat{i} - y\hat{j} - 4z\hat{k}$$

at point $(1,2,1)$

$$\nabla\times B = 5\hat{i} - 2\hat{j} - 4\hat{k}.$$

(iv) Grad. $\delta\cdot\nabla A = \frac{\delta(\nabla\cdot A)\, i}{\delta x} + \frac{\delta(\nabla\cdot A)}{\delta y}\hat{j} + \frac{\delta(\nabla\cdot A)\hat{k}}{\delta z}$

where $\nabla\cdot A = 2xy + (x+z) + 2xz.$

$$\nabla\cdot(\nabla\cdot A) = (2y+1+2z)\hat{i} + (2x)\hat{j} + (1+2x)\hat{k}$$

at point $(1,2,1)$.

$$\nabla(\nabla\cdot A) = 2\hat{i} + 2\hat{j} + 3\hat{k}.$$

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(v) Curl $A = \nabla \times (\nabla \times A)$ let $\nabla \times A = C$.

$$\nabla \times C = \left(\frac{\delta c_z}{\delta y} - \frac{\delta c_y}{\delta z}\right)\hat{i} - \left(\frac{\delta c_z}{\delta x} - \frac{\delta c_x}{\delta z}\right)\hat{j} + \left(\frac{\delta c_y}{\delta x} - \frac{\delta c_x}{\delta y}\right)k$$

$$\nabla \times A = \left(\frac{\delta a_z}{\delta y} - \frac{\delta a_y}{\delta z}\right)\hat{i} - \left(\frac{\delta a_z}{\delta x} - \frac{\delta a_x}{\delta z}\right)\hat{j} - \left(\frac{\delta a_y}{\delta x} - \frac{\delta a_x}{\delta y}\right)k$$

$$A = x^2 y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}.$$

$$\frac{\delta a_z}{\delta y} = 0, \quad \frac{\delta a_y}{\delta z} = y, \quad \frac{\delta a_x}{\delta x} = z^2.$$

$$\frac{\delta a_x}{\delta z} = 0, \quad \frac{\delta a_y}{\delta x} = y, \quad \frac{\delta a_x}{\delta y} = x^2$$

$$\therefore \nabla \times A = (0 - y)\hat{i} - (z^2 - 0)\hat{j} + (y - -x^2)R.$$

$$\nabla \times A = -y\hat{i} - z^2\hat{j} + (y - x^2)k$$

where $\nabla \times A = C$.

$$\frac{\delta c_z}{\delta y} = 1, \quad \frac{\delta c_y}{\delta z} = -2z, \quad \frac{\delta c_x}{\delta x} = -2x.$$

$$\frac{\delta c_x}{\delta z} = 0, \quad \frac{\delta c_y}{\delta x} = 0, \quad \frac{\delta c_x}{\delta y} = -1$$

$$\therefore \nabla \times C = (1 - (-2x))\hat{i} - (-2x - 0)\hat{j} + (0 - (-1))k$$

$$\nabla \times C = (1 + 2z)\hat{i} + 2x\hat{j} + \hat{k}$$

$\therefore$ At point $(1, 2, 1)$

$$\nabla \times C = \nabla \times (\nabla \times A) = (1 + 2)\hat{i} + 2\hat{j} + \hat{k}.$$

$$\therefore \nabla \times (\nabla \times A) = 3\hat{i} + 2\hat{j} + \hat{k}$$