

17/ENG03/020

CIVIL ENGINEERING

ENG 281

## Question 1

Given a function to be as in equation (1)

$$f(x) = \pi$$

Find  $\lim_{x \rightarrow 3} f(x)$

$$\lim_{x \rightarrow 3}$$

Soln  ~~$f(x) = \pi$~~

$$f(x) = \pi$$

$$\lim_{x \rightarrow 3} = \pi$$

## Question 2

The model of a system has been developed by an engineer to be as given in equation (2)

$$f(x) = 5x - 21$$

Given that  $\delta = 0.1$ , and using a step of 0.01, demonstrate in tabular form, that the limit of the model as  $x \rightarrow 6$  is equal to 9.

Soln

$$\delta = 0.01$$

$$x = 6$$

n	<del><math>n \cdot \delta</math></del> $n \cdot \delta$	$x - n \cdot \delta = v$	$f(x) \quad x \rightarrow v$
0	0	$6 - 0 = 6$	9
1	0.01	$6 - 0.01 = 5.99$	8.95
2	0.02	$6 - 0.02 = 5.98$	8.9
3	0.03	$6 - 0.03 = 5.97$	8.85
4	0.04	$6 - 0.04 = 5.96$	8.8
5	0.05	$6 - 0.05 = 5.95$	8.75
6	0.06	$6 - 0.06 = 5.94$	8.7
7	0.07	$6 - 0.07 = 5.93$	8.65
8	0.08	$6 - 0.08 = 5.92$	8.6
9	0.09	$6 - 0.09 = 5.91$	8.55
10	0.10	$6 - 0.10 = 5.9$	8.5

$n$	$n \cdot \sigma$ <del><math>A = \sigma</math></del>	$x + n \cdot \sigma = v$	$f(x)$ $x \rightarrow v$
0	0	$6 + 0 = 6$	9
1	0.01	$6 + 0.01 = 6.01$	9.05
2	0.02	$6 + 0.02 = 6.02$	9.1
3	0.03	$6 + 0.03 = 6.03$	9.15
4	0.04	$6 + 0.04 = 6.04$	9.2
5	0.05	$6 + 0.05 = 6.05$	9.25
6	0.06	$6 + 0.06 = 6.06$	9.3
7	0.07	$6 + 0.07 = 6.07$	9.35
8	0.08	$6 + 0.08 = 6.08$	9.4
9	0.09	$6 + 0.09 = 6.09$	9.45
10	0.10	$6 + 0.10 = 6.1$	9.5

### Question 3

Find the limit of the model equation given in equation

$$(3) \quad \lim_{x \rightarrow 3^+} \frac{3-x}{|3-x|}$$

Soln

$$\lim_{x \rightarrow 3^+} = \frac{3 - (3+h)}{|3 - (3+h)|} = \frac{h}{|h|} = 1$$

$$\lim_{x \rightarrow 3^-} = \frac{3 - (3-h)}{|3 - (3-h)|} = \frac{-h}{|-h|} = -1$$

$$\therefore \lim_{x \rightarrow 3^+} \frac{3-x}{|3-x|} \neq \lim_{x \rightarrow 3^-} \frac{3-x}{|3-x|}$$

### Question 4

Evaluate the limit of the model given in equation (4), if it exists.

$$\lim_{x \rightarrow 3} \frac{x-3}{|x-3|}$$

Soln

$$\lim_{x \rightarrow 3} \frac{3-3}{3-3} = \frac{0}{0}$$

Differentiate

$$\frac{x-3}{|x-3|} = \frac{1}{1}$$

$$\lim_{x \rightarrow 3} = \frac{1}{1} = 1$$

### Question 5

Show that the function given in equation (5)

$$f(x) = \sqrt{x-4}$$

is continuous on the interval  $[4, 8]$

Soln

$$f(x) = \sqrt{x-4}$$

$$= (x-4)^{\frac{1}{2}}$$

$$y = (x-4)^{\frac{1}{2}}$$

$$\text{Let } u = x - 4$$

$$y = u^{\frac{1}{2}}$$

$$\frac{du}{dx} = 1$$

$$\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 1 \cdot \frac{1}{2} u^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot (x-4)^{-\frac{1}{2}}$$

$$= \frac{1}{2(x-4)^{\frac{1}{2}}}$$

$$= \frac{1}{2} \sqrt{x-4}$$