30/9/2018

Assignment

1) Given a function to be as in Eqn (1)

$f(x) = \pi$ ------(1)

find $\lim_{x \to 3} f(x)$

Soln

$$\lim_{x \to 3} \{f(x)\} = \lim_{x \to 3} \pi$$

$$= \pi$$

2) The model of a system has been developed by an Engineer to be as given in Equation (2)

$f(x) = 5x - 21$ ------ (2)

Given the $\delta = 0.1$, and using a step of 0.01, demonstrate, in tabular form, that the limit of the model as $x \to 6$ is equal to 9.

Soln

$\cancel{f(x)}$ $\cancel{f(x) = 5x - 21}$

$\cancel{9+\delta}$ 6 $\cancel{9-\delta}$ $\cancel{f(x)}$

Soln

$f(x) = 5x - 21$

$x \to$				$\leftarrow x$
$f(x)$	$a - \delta$	6	$a + \delta$	$f(x)$
8.50	5.9		6.1	9.50
8.55	5.91		6.09	9.45
8.60	5.92		6.08	9.40
8.65	5.93		6.07	9.35
8.70	5.94		6.06	9.30
8.75	5.95		6.05	9.25
8.80	5.96		6.04	9.20
8.85	5.97		6.03	9.15
8.90	5.98		6.02	9.10
8.95	5.99		6.01	9.05
9.00	6.00		6.00	9.00

$\therefore \lim_{x \to 6} f(x) = \lim_{x \to 6} (5x - 21) = 9$

Therefore, $\lim_{x \to 6} (5x - 21)$ is a both sided limit and exists on the left hand side and Right hand side. The limit is defined on both sides.

8.70	5.94	6.06	9.30
8.75	5.95	6.05	9.25
8.80	5.96	6.04	9.20
8.85	5.97	6.03	9.15
8.90	5.98	6.02	9.10
8.95	5.99	6.01	9.05
9.00	6.00	6.00	9.00

$$\therefore \quad \lim_{x \to 6} f(x) = \lim_{x \to 6} (5x - 21) = 9$$

Therefore, $\lim_{x \to 6} (5x - 21)$ is a both sided limit and exists on the left hand side and Right hand side. The limit is defined on both sides.

3) Find the limit of the model equation given in Equation (3)

$$\lim_{x \to 3^+} \frac{3-x}{|3-x|} \quad \text{———} \quad (3).$$

<u>Soln</u>

$$\lim_{x \to 3^+} \frac{3-x}{|3-x|} = \lim_{\delta \to 0} \frac{\cancel{3}-\delta+\cancel{3}}{|\cancel{3}-\delta+\cancel{3}|}$$

$$= \lim_{\delta \to 0} \frac{-\delta}{|-\delta|} = -1.$$

$$\frac{-1}{0.1} = -1$$

4) Evaluate the limit of the model given in Equation (4), if it exists.

$$\lim_{x \to 3} \frac{x-3}{|x-3|} \quad \text{———} \quad (4)$$

Soln

$$\lim_{x \to 3^+} \frac{x-3}{|x-3|} = \lim_{\delta \to 0} \frac{\cancel{3}+\delta-\cancel{3}}{|\cancel{3}+\delta-\cancel{3}|}$$

$$= \lim_{\delta \to 0} \frac{\delta}{|\delta|} = \underline{1}$$

$$\lim_{x \to 3^-} \frac{x-3}{|x-3|} = \lim_{\delta \to 0} \frac{\cancel{3}-\delta-\cancel{3}}{|\cancel{3}-\delta-\cancel{3}|}$$

$$= \lim_{\delta \to 0} \frac{-\delta}{|-\delta|} = \underline{-1}$$

Therefore; $\lim_{x \to 3} \frac{x-3}{|x-3|}$ does not exist, since the left-hand limit and right-hand limit is not approximately the same.

5) Show that the function given in Equation (5)

$$f(x) = \sqrt{x-4} \quad \text{———} \quad (5)$$

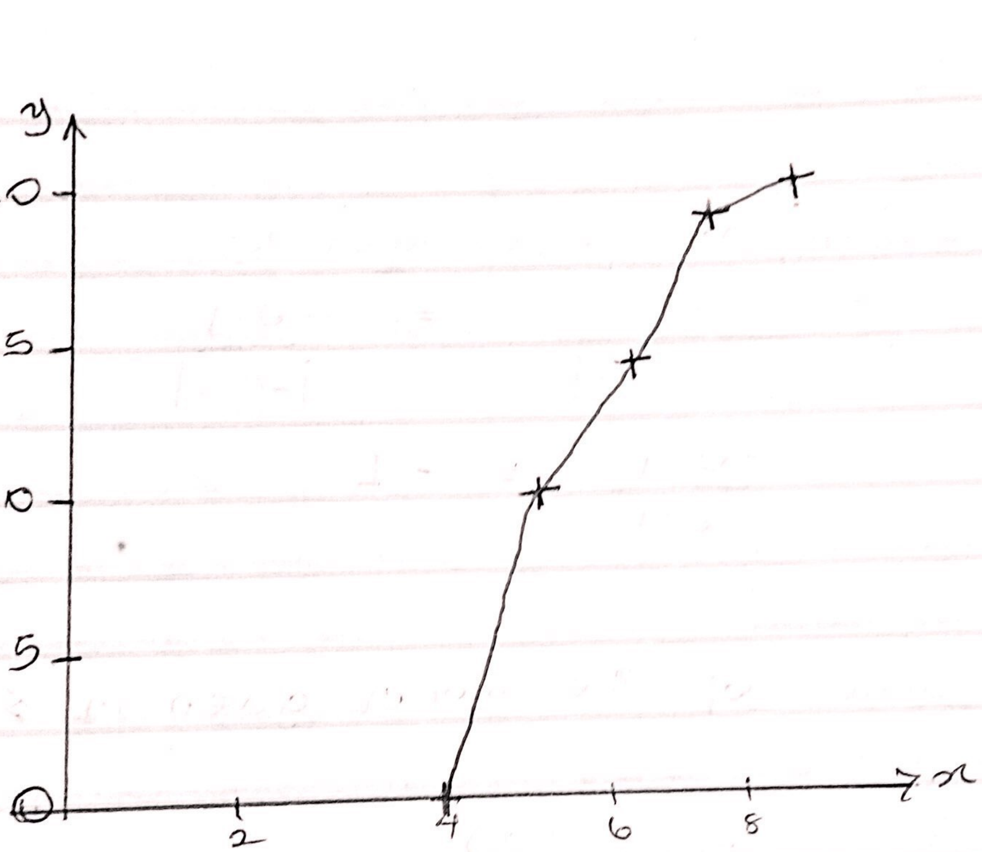
is continuous on the interval $[4, 8]$

Soln

Sketch a graph for $f(x) = \sqrt{x-4}$.

TABLE OF VALUES

x	4	5	6	7	8
y	0.0	1.0	1.4	1.7	2.0



from the graph; $f(x) = \sqrt{x-4}$ is continuous on the interval $[4, 8]$.