

Assignment

⇒ (1) Given a function to be as in Equation (1)

$$f(x) = \pi, \text{ find } \lim_{x \rightarrow 3} f(x)$$

Sol

$$f(x) = \pi = \pi(x^0)$$

$$\lim_{x \rightarrow 3} = \pi(3^0) = \pi = 3.142$$

⇒ (2) The model of a system has been developed by an Engineer to be as

given in equation (2) $f(x) = 5x - 21$

Given that $\delta = 0.1$ and using a step of 0.01, demonstrate in tabular form that the limit of the model as $x \rightarrow 6$ is equal to 9.

Sol

using the equation $5x - 21$

∴ as $x \rightarrow 6$, $5x - 21 = 5(6) - 21 = 30 - 21 = 9$

x	f(x) = 5x - 21	y
5.91	5(5.91) - 21 = 8.55	8.55
5.92	5(5.92) - 21 = 8.6	8.6
5.93	5(5.93) - 21 = 8.65	8.65
5.94	5(5.94) - 21 = 8.7	8.7
5.95	5(5.95) - 21 = 8.75	8.75
5.96	5(5.96) - 21 = 8.8	8.8
5.97	5(5.97) - 21 = 8.85	8.85
5.98	5(5.98) - 21 = 8.9	8.9
5.99	5(5.99) - 21 = 8.95	8.95
6	5(6) - 21 = 9	9

∴ f(x) of $5x - 21$ as $x \rightarrow 6$ is equal to 9

⇒ (3) Find the limit of the model equation given in equation 3

$$\lim_{x \rightarrow 3^+} \frac{3-x}{|2-x|}$$

Q.17

$$\lim_{x \rightarrow 3^+} \frac{3-x}{|3-x|} = \lim_{x \rightarrow 3^+} \frac{(3+h)-3}{|(3+h)-3|} = \frac{3+h-3}{|3+h-3|}$$

$$\lim_{h \rightarrow 0} \frac{3-(3+h)}{|3-(3+h)|} = \frac{3-(3+h)}{|3-(3+h)|} = \frac{-h}{h} = -1$$

(4) Evaluate the limit of the model given in equation 4, if it exists.

$$\lim_{x \rightarrow 3} \frac{x-3}{|x-3|}$$

Sol

$$\lim_{x \rightarrow 3} \frac{x-3}{|x-3|} = \frac{3-3}{|3-3|} = \frac{0}{0} = \text{undefined.}$$

$$\therefore \lim_{x \rightarrow 3^+} \frac{x-3}{|x-3|} = \lim_{h \rightarrow 0} \frac{(3+h)-3}{|(3+h)-3|} = \frac{h}{h} = 1$$

$$\lim_{x \rightarrow 3^-} \frac{x-3}{|x-3|} = \lim_{h \rightarrow 0} \frac{(3-h)-3}{|(3-h)-3|} = \frac{-h}{|-h|} = \frac{-h}{h} = -1$$

$$\therefore \lim_{x \rightarrow 3^+} \frac{x-3}{|x-3|} \neq \lim_{x \rightarrow 3^-} \frac{x-3}{|x-3|}$$

(5) Show that the function given in equation 5 $f(x) = \sqrt{x-4}$ is continuous on the interval $[4, 8]$.

Sol

Given the function $f(x) = \sqrt{x-4}$

$$\therefore \text{as } x \rightarrow 4, f(x) = \sqrt{4-4} = \sqrt{0} = 0 \quad \text{--- (1)}$$

$$\text{and } x \rightarrow 8, f(x) = \sqrt{8-4} = \sqrt{4} = 2 \quad \text{--- (2)}$$

\therefore from eqs (1) and (2), the value of $\sqrt{x-4}$ is continuous at $x=8$