

Given the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$  find  $\det(A)$ .  
 $\det(A) = 1(15-12) - 2(5-12) + 3(10-9) = 3 + 14 + 3 = 20$   
 The matrix  $A$  is invertible since  $\det(A) \neq 0$ .  
 $A^{-1} = \frac{1}{\det(A)} \text{adj}(A) = \frac{1}{20} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & 4 \\ 3 & -4 & 1 \end{pmatrix}$

(3) The matrix  $A$  is a system of linear equations. To solve for  $x, y, z$  we use Cramer's rule.  
 $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$ ,  $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$   
 $\det(A) = 20$   
 $\det(A_x) = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 20$   
 $\det(A_y) = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 2 & 4 \\ 3 & 3 & 5 \end{vmatrix} = 0$   
 $\det(A_z) = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 4 \end{vmatrix} = 0$   
 $x = \frac{\det(A_x)}{\det(A)} = 1$ ,  $y = \frac{\det(A_y)}{\det(A)} = 0$ ,  $z = \frac{\det(A_z)}{\det(A)} = 0$

(3) Find the rank of the matrix  $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$ .  
 $\det(A) = 20 \neq 0$ , so the rank is 3.

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