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$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Hence: general solution

$$x = C.F + P.I$$

Finding complementary function:

We assume $f(x) = 0$

$$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

m factorizing

$$m^2 + 2m + 3m + 6 = 0$$

$$m(m+2) + 3(m+2)$$

$$(m+3) = 0 \quad \text{or} \quad (m+2) = 0$$

$$\therefore m = -3 \text{ or } -2$$

$$\therefore x = Ae^{-2t} + Be^{-3t}$$

Finding the particular integral

$$x = A\cos t + B\sin t$$

$$\frac{dx}{dt} = -A\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$$

Substituting x, dx/dt and d^2x/dt^2

$$-A\cos t - B\sin t + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$$

$$(-A + 5B + 6A)\cos t + (-B - 5A + 6B)\sin t = \cos t$$

Equating co-efficients

$$-A + 5B + 6A = 1 \quad \text{---(i)}$$

$$-5A + 5B = 0 \quad \text{---(ii)}$$

from equ (ii)

$5A = 5B$

$\therefore$ A = dividing both sides by 5

$\frac{5A}{5} = \frac{5B}{5}$

$A = B$.

Substituting $A = B$ into equ (i)

$-B + 5B + 6B = 1$

$10B = 1$

$\therefore B = \frac{1}{10}$

$\therefore A = \frac{1}{10}$

$\therefore P.I. x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$.

$\therefore$ general Solution:

$x = C.F + P.I$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$

given initial conditions

$t = 0$ when $x = 0.1$

$\therefore 0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos 0 + \sin 0)$

$0.1 = A + B + \frac{1}{10}(1)$

$A + B = \frac{0.1}{1} - \frac{1}{10}$

$A + B = \frac{1 - 1}{10} = 0$

$\therefore A + B = 0$ -- (i)

also given

when $t = 0$, $x = 0.1$ and $dx/dt = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10}(-\sin t + \cos t) = 0$

$= -2Ae^{-2(0)} - 3Be^{-3(0)} + \frac{1}{10}(-\sin 0 + \cos(0)) = 0$

$-2A - 3B + \frac{1}{10}(1) = 0$

$-2A - 3B = -\frac{1}{10}$

from equ (i)

$A + B = 0$

$\therefore \quad A = -B$

Substituting $A = -B$ into eqn (1)

$$-2(-B) - 3B = -\frac{1}{10}$$

$$2B - 3B = -\frac{1}{10}$$

$$-B = -\frac{1}{10}$$

multiplying both sides by -1

$$B = \frac{1}{10}$$

$$\therefore \quad A = -\left(\frac{1}{10}\right)$$

Particular Solution:

$$x = \frac{-1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + 1/10 (\cos t + \sin t)$$

$$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t).$$

Steady-State Solution

$$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t)$$

the transient part of the equation will not be considered since we are dealing with the steady state

$$\therefore x = \frac{1}{10}(\cos t + \sin t)$$

$\therefore$ ~~dx~~ we know $\frac{dx}{dt} = 0$ for steady state

$$\therefore \frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$$

$$\therefore -\sin t + \cos t = 0$$

$$\therefore \cos t = \sin t$$

$$\therefore t = 45^\circ.$$

$$\therefore x = \frac{1}{10}(\cos 45 + \sin 45)$$

$$x = \frac{1}{10}(\cos 45 + \sin 45)$$

Recall (Sinusodial expression)

$$A\cos\omega t + B\sin\omega t = K\cos(\omega t - \theta)$$

But

$$\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$$

where

$$K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$$

$\theta = 0^\circ$ (since its in the same phase)

$\therefore$

$$x = \frac{1}{10}(\cos 45 + \sin 45) = 0.14 \sin(45 + 90^\circ)$$

$$x = 0.14 \sin(45 + 90^\circ).$$

MATLAB.

~~Command window~~ Commandwindow

```
Clear all
clc
syms t
x = 0.1*(exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
tn = [0: 0.01: 15]
xn = subs(x, tn)
plot(tn, xn)
grid on
grid minor
xlabel('time (sec)')
ylabel('Amplit Vibration')
axis tight
```