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16/ENG01/ÚK
ENG 381
CHEMICAL ENGINEERING

① $d^2x/dt^2 + 5dx/dt + 6x = \cos t$

Assuming $\cos t = 0$

$$\frac{d^2x}{dt^2} + \frac{5dx}{dt} + 6x = \cos t$$

$a^2 + 5a + 6 = 0$

$a^2 + 3a + 2a + 6 = 0$

$(a+2)(a+3) = 0$

$a = 2 ; a = -3$

Complementing Integration

$x = Ae^{-2t} + Be^{-2t}$

Particular Integral

$f(x) = \cos t, \; x = C\cos t + \Delta \sin x$

$dx/dt = -C\sin t + \Delta \cos t$

$d^2x/dt^2 = -(C\cos t - \Delta \sin t)$

Substituting

$-(C\cos t - \Delta \sin t + 5C - C\sin t + \Delta \cos t) + 6(C\cos t + \Delta \sin t)$

$= \cos t$

$-(C\cos t - \Delta \sin t - 5C\sin t + 5\Delta \cos t + 6C\cos t + 6\Delta \sin t$

$\cos t(-C + 5\Delta + 6C) + 6\sin t(-\Delta - 5 + 6\Delta) = \cos t$

$\cos t(5C + 5\Delta) + \sin t(5D - 5t) = \cos t$

By Comparing Co-efficients

$5\Delta + 5C = 1$

$5\Delta - 5C = 0$

$10D = 1$

$D = \frac{1}{10}$

$3D + 5C = 1$

$5 \cdot \frac{1}{10} + 5C = 1$

$5C + 0.5 = 1$

$5C = 0.5$

$C = \frac{0.5}{5} = \frac{1}{10}$

$C = \frac{1}{10} \quad D = \frac{1}{10}$

$f(x) = \frac{1}{10} \cos t + \frac{1}{10} \sin t$

General solution = $Cf + PI$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t.$

when $t = 0$; $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$

$0.1 = A \cdot 1 + B \cdot 1 + \frac{1}{10} \cdot 0 + \frac{1}{10} \cdot 1$

$0.1 = A + B + 0.1$

$A + B = 0$

$A = -B$

$t = 0 \quad \frac{dx}{dt} = 0.$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$

$0 = -2 \cdot 1 - 3 \cdot 1 - \frac{1}{10} \sin 0 + \frac{1}{10} \cos 0$

$0 = -2A - 3B + 0.1$

$2A + 3B = 0.1 \quad \cdots \quad (x)$

from eq 3.

$A = -B$

Sub $-B$ for A in eqn x

$-2B + 3B = 0.1$

$B = 0.1$

$A = -B$

$A = -0.1$

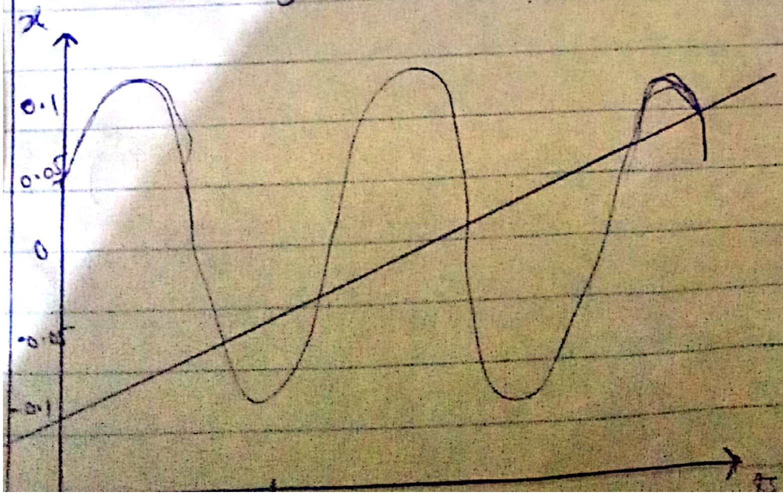
Particular solution

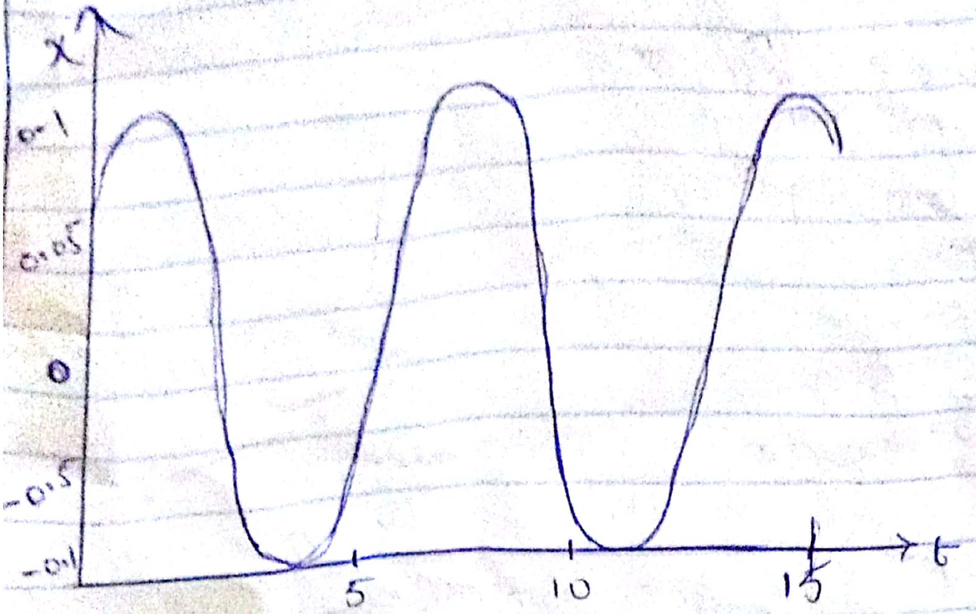
$$x = -0.1e^{-2t} + 0.1e^{3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t.$$

$$x = -\frac{e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{1}{10}\cdot\cos t + \frac{\sin t}{10}$$

ii) MATLAB.

```
commandwindow
clear
clc
closeall
syms t
x(t) = 0.1*((exp(-3*t)) - (exp(-2*t)) + cos(t) + sin(
tn = [0:0.01:15]
xd = subs(x(t), tn)
figure(1)
plot(tn, xd)
~~grid~~ grid minor
grid on
axis tight.
```





steady state form of $x = K\sin(t+a)$

ii) The steady state is $x = C\cos t + D\sin t$

Assume $C = K\cos a$ and $D = K\sin a$

$x = (K\cos a)\sin t + (K\sin a)\cos t$

$x = K(\cos a \cdot \sin t + \sin a \cdot \cos t)$

$x = K\left[\cos\left(\frac{\pi}{2} - t\right)\cos(-a) - \sin\left(\frac{\pi}{2} - t\right)\sin(-a)\right]$

$x = K\left[\cos\left(\left(\frac{\pi}{2} - t\right) - a\right)\right]$

$x = K\left[\cos\frac{\pi}{2} - (t+a)\right]$

$x = K\left(\cos\frac{\pi}{2}\cos(t+a) + \sin\frac{\pi}{2}\sin(t+a)\right)$

$x = K(0 \times \cos(t+a) + 1 \times \sin(t+a))$

$x = K[\sin(t+a)]$

The steady state solution $x(C\cos t + D\sin t)$ can be written in the form $x = K(t+a)$