Akintimehin Damola
16/ENG06/009
ENG 381

Assignment

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

assuming $\cos t \neq 0$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$\frac{dx}{dt} = q$

$q^2 + 5q + 6 = 0$

$q^2 + 3q + 2q + 6 = 0$

$q(q+3) + 2(q+3) = 0$

$(q+2)(q+3) = 0$

$q = -2, \quad q = -3$

complementing integration

$x = Ae^{-2t} + Be^{-2t}$

particular integral = P.I

$f(x) = \cos t, \quad x = C\cos t + D\sin t$

~~$C\cos t - D\sin t + 5(-C\sin t + D\cos t$~~

$\frac{dx}{dt} = -C\sin t + D\cos t$

$$\frac{d^2x}{dt^2} = -(C\cos t - D)\sin t$$

Substituting

$-(C\cos t - D)\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$
$= \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

By comparing coefficients

$5D + 5C = 1$ ------ (1)

$5D - 5C = 0$ ---- (2)

Simultaneously

$10D = 1$, $D = \frac{1}{10}$

$5D + 5C = 1$

Sub in equ (1)

$5 \times \frac{1}{10} + 5C = 1$

$\frac{1}{2} + \frac{5C}{1} = 1$ $\quad \frac{1 + 10C}{2} = 1$

$1 + 10C = 2$ $\quad 10C = 1$

$C = \frac{1}{10}$

$D = \frac{1}{10}$

~~where~~ $f(x) = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

General solution = $C_f + PI$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

when $t = 0$, $x = 0.1$

$0.1 = Ae^{-2(0)} + Be^{3(0)} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$0.1 = A \cdot 1 + B \cdot 1 + \frac{1}{10} \cdot 0 + \frac{1}{10} \cdot 1$

$0.1 = A + B + 0.1$

$A + B = 0$

$A = -B$

$t = 0$, $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -2 \cdot 1 - 3 \cdot 1 - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$

$0 = -2A - 3B + 0.1$

$2A + 3B = 0.1 \quad \ldots\ldots (x)$

From eq

$A = -B$

Sub $-B$ for A in eqn x

$-2B + 3B = 0.1$

$B = 0.1$

$A = -B$

$A = -0.1$

Particular Solution

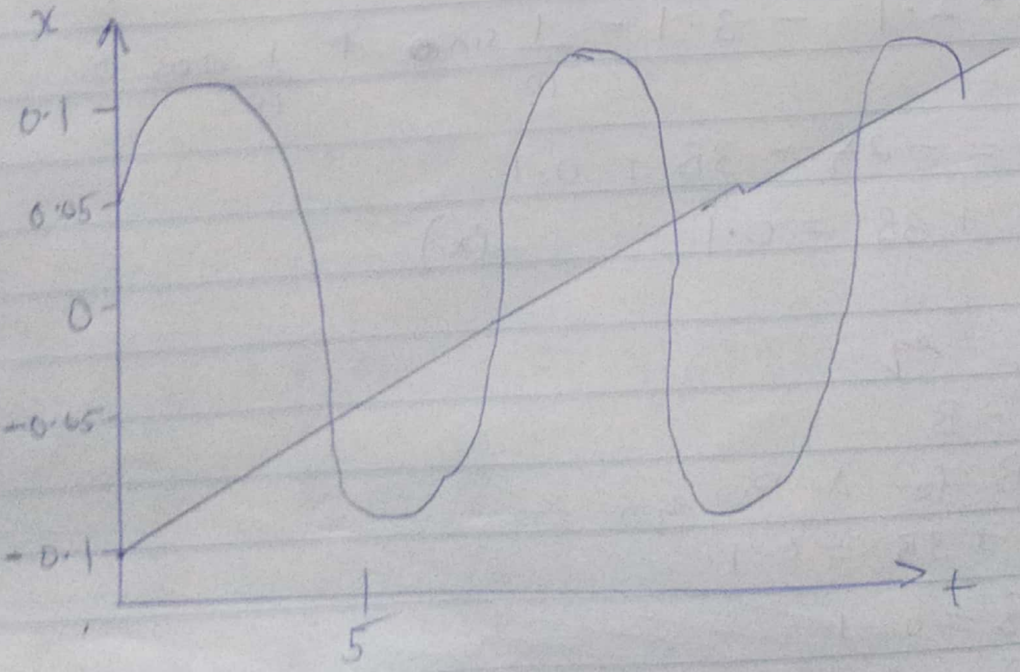
$x = -0.1e^{-2t} + 0.1e^{3t} + \frac{1}{16}\cos t + \frac{1}{10}\sin t$

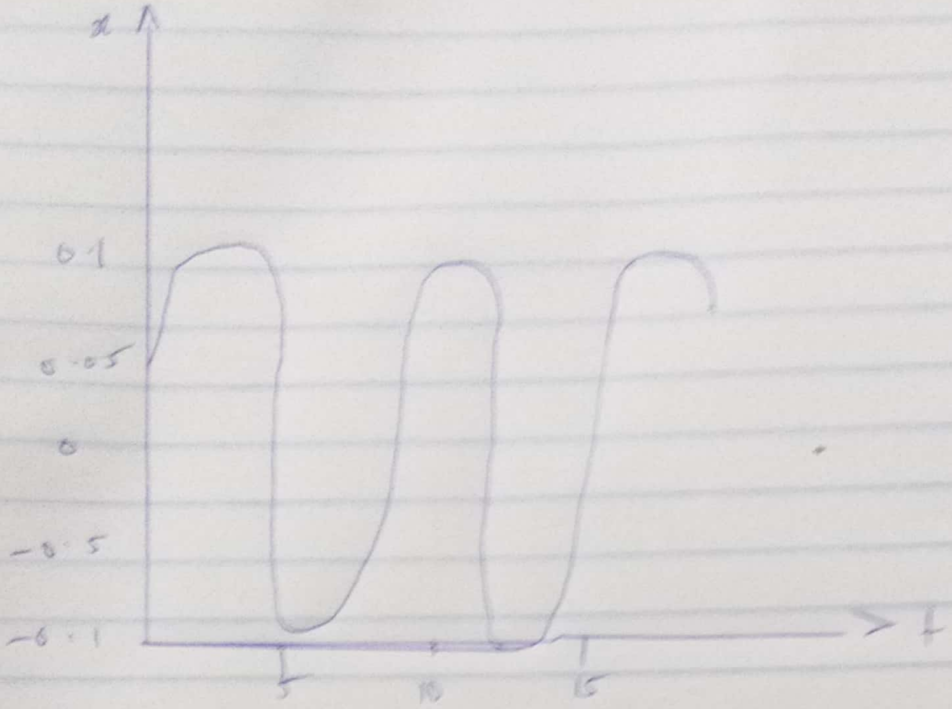
$$x = -\frac{e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{1}{10}\cos t + \frac{\sin t}{10}$$

ii) MATLAB

Command window

```
clear
clc
close all
syms t
x(t) = 0.1*((exp(-3*t)) - ((exp(-2*t)) + cos(t) + sin(t)
tn = [0:0.01:15]
xd = subs(x(t), tn)
figure(1)
plot(tn, xd)
grid minor
grid on
axis tight
```





Steady state form of $x = k\sin(t + a)$

ii) The steady state is $x = C\cos t + D\sin t$

Assume $C = k\cos a$ and $D = k\sin a$

$x = (k\cos a)\sin t + (k\sin a)\cos t$

$x = k(\cos a \cdot \sin t + \sin a \cdot \cos t)$

$x = k\left[\cos\left(\frac{\pi}{2} - t\right)\cos(-a) - \sin\left(\frac{\pi}{2} - t\right)\sin(-a)\right]$

$x = k\left[\cos\left(\left(\pi/2 - t\right) - a\right)\right]$

$x = k\left[\cos \frac{\pi}{2} - (t + a)\right]$

$x = k\left[\cos\frac{\pi}{2}\cos(t + a) + \sin\frac{\pi}{2}\sin(t + a)\right]$

$x = k(0 \times \cos(t + a) + 1 \times \sin(t + a))$

$x = k[\sin(t + a)]$

The steady state solution $x(C\cos t + D\sin t)$ can be written in the form $x = k(t + a)$.

Scanned by CamScanner