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16/ENGG07/008

PETROLEUM ENGINEERING

ENGG 381

ASSIGNMENT 1

1. $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

assuming $\cos t = 0$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+2)(m+3) = 0$$

$$m_1 = -2 \quad \text{or} \quad m_2 = -3$$

Complementary function $x = Ae^{-2t} + Be^{-3t}$

Particular Integration

$$f(x) = \cos t \quad x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t + (-D \sin t)$$

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t$$

$$\cos t (-C + 5D + 6C) + \sin t (-D - 5C + 6D) = \cos t$$

$$\cos t (5C + 5D) + \sin t (5D - 5C) = \cos t$$

By comparing coefficients

$$5D + 5C = 1 \quad \text{--- (1)}$$

$$5D - 5C = 0 \quad \text{--- (2)}$$

by adding eq 1 and 2

$$10\Delta = 1$$

$$\Delta = \frac{1}{10}$$

Subs. $\frac{1}{10}$ for Δ in eq 1

$$5\Delta + 5C = 1$$

$$\frac{5}{10} + 5C = 1$$

$$5C = 1 - 0.5$$

$$5C = 0.5$$

$$C = \frac{0.5}{5} = \frac{1}{10}$$

∴ Particular Integration

$$f(t) = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

General solution = Cf + P.I

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

when $t=0$, $x=0.1$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos 0 + \frac{1}{10} \sin 0$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0.1 - 0.1$$

$$A + B = 0$$

$$A = -B \quad \text{--- (3)}$$

when $t=0$ $\frac{dx}{dt} = 0$

$$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10} \sin 0 + \frac{1}{10} \cos 0$$

$$0 = -2A - 3B + 0.1$$

$$2A + 3B = 0.1 \quad \text{--- (4)}$$

from eq 3

$$A = -B$$

subs $-B$ for A in eq 4

$$-2B + 3B = 0.1$$

$$B = 0.1$$

$$A = -B$$

$$\therefore A = -0.1$$

Particular solution

$$x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

OR

$$x = \frac{-e^{-2t}}{10} + \frac{e^{-3t}}{10} + \frac{1}{10} (\cos t + \sin t)$$

MATLAB

Command window

clear

clc

close all

syms t

$$x(t) = 0.1 * ((\exp(-3 * t)) - (\exp(-2 * t)) + \cos(t) + \sin(t))$$

$$tn = [0 : 0.01 : 15]$$

$$xd = \text{subs}(x(t), tn)$$

figure(1)

plot(tn, xd)

axis tight

grid on

grid minor

