Name: Thank God Clinton.
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Course: ENG 381
Department: Mechanical Engineering.

(1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

solution.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

Using Auxiliary Equation

$$m^2 + 5m + 6 = 0$$

~~Using~~ $m_1 = -2, \quad m_2 = -3.$

$\therefore \; x = Ae^{m_1 t} + Be^{m_2 t}$

$$x = Ae^{-2t} + Be^{-3t} = C.F$$

$P.I = f(t) = \cos t$

$$x = A\cos t + B\sin t$$

$$\frac{dx}{dt} = -A\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$$

$$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6 = \cos t$$

$$\Rightarrow (-A\cos t - B\sin t) + 5(-A\sin t + B\cos t) + 6(A\cos t + B\sin t) = \cos t$$

$$[-A\cos t - B\sin t) + (-5A\sin t + 5B\cos t) + (6A\cos t + 6B\sin t) = \cos t$$

$$(-A\cos t + 6A\cos t - B\sin t - 6B\sin t - 5A\sin t + 5B\cos t = \cos t.$$

Collecting the coefficient of like terms

$$5A + 5B = 1$$
$$-5A + 5B = 0$$
$$10B = 1$$

$$B = \frac{1}{10}$$

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$\therefore 5A + 5B = 1$

$$5A + 5\left(\frac{1}{10}\right)_2 = 1$$

$$5A + \frac{1}{2} = 1$$

$$5A = 1 - \frac{1}{2}$$

$$10A = 1$$

$$A = \frac{1}{10}$$

$$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$P.I = \frac{1}{10}(\cos t + \sin t)$$

$$x = C.F + P.I$$

$$x = \frac{1}{10}e^{-3t} + \frac{1}{10}e^{-2t} + \frac{1}{10}[\cos t + \sin t].$$

(ii) Write a matlab program to plot the relationship between x if for $0 \leq t \leq 15$ unit using a step size of 0.01 unit.

Solution.

```
- Command window
- Clear
- Clc
- Close all
- syms t
x = 0.1*(exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
  tn = [0; 0.01 : 15]
  xn = subs(x, tn)
  figure(1)
plot(tn, xn)
grid on
```

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grid minor
axis tight
x label ('time')
y label ('vibrations')

(iii) Write the steady state solution of the system in form of $x = k\sin(t+a)$

Solution.

$$x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \sin t + \cos t\right)$$

at steady state $\frac{dx}{dt} = 0$ i.e

change in x with time is zero

$$\therefore \frac{dx}{dt} = \frac{1}{10}\left(-3e^{-3t} - e^{-2t} + \cos t - 3\sin t\right)$$

nb: the exponentials result zero

$$0 = \cos t - \sin t$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$x = \frac{1}{10}\left(\cos 45 + \sin 45\right) = \frac{\sqrt{2}}{10}$$

From sinosodial expression

$$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$$

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where; $k = \sqrt{A^2+B^2} = \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{\frac{1+1}{10^2}}$

$$k = \frac{\sqrt{2}}{10}$$

$\theta = 0^\circ$ (since its in same phase)

Recall $x = k\sin(t+a)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45+a)$$

$$1 = \sin(45+a)$$

$$45+a = \sin^{-1}(1)$$

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$$\theta = 90 - 45 = 45^\circ \simeq \pi/4$$

The steady state solution is ;

$$x = \frac{\sqrt{2}}{10}\left(\sin 1 + x/4\right).$$

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