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i $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

using the Auxillery Eqn

$m^2 + 5m + 6 = 0$

$m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$

$(m+3)(m+2)$

$m_1 = -3, \; m_2 = -2$

$\therefore \; CF = x_c = Ae^{-3t} + Be^{-2t}$

P.I $x_p = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore (-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t + 0$

$5D + 5C = 1$

$-5C + 5D = 0$

$5D = 5C$

$D = C$

$+5(C) + 5C = 1$

$5C + 5C = 1$

$10C = 1$

$C = \frac{1}{10} \quad \therefore C = D = \frac{1}{10}$

$$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

General soln $\Rightarrow x = Ae^{-3t} + Be^{-2t} + \frac{\cos t}{10} + \frac{\sin t}{10}$

$$x = Ae^{-3t} + De^{-2t} + \frac{1}{10}(\cos t + \sin t)$$

when $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$$0.1 = Ae^{0} + Be^{0} + \frac{1}{10}(\cos 0 + \sin 0)$$

$$0.1 = A + B + \frac{1}{10}(1 + 0)$$

$$A + B = 0 \quad \therefore A = -B \qquad \text{--- eqn } *$$

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}(-\sin t + \cos t)$$

$\frac{dx}{dt} = 0$, $t = 0$

$$0 = -3Ae^{0} - 2Be^{0} + \frac{1}{10}(-\sin(0) + \cos(0))$$

$$0 = -3A - 2B + \frac{1}{10}(0 + 1)$$

$$3A + 2B = 0.1$$

$$3A + 2B = \frac{1}{10}$$

from eqn $*$ $A = -B$

$$3(-B) + 2B = 0.1$$

$$-3B + 2B = 0.1$$

$$-B = 0.1$$

$$x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$$

III Steady State Solution

$$x = \frac{1}{10}\left(-e^{-2t} + e^{-3t}\right) + (\cos t + \sin t)$$

The transient Point of the equation will not be considered Since we are dealing with the steady state

we know $\frac{dx}{dt} = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$$\therefore t = 45°$$

$$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

from Sinusoidal expression

$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90°)$

where

$$k = \sqrt{A^2 + B^2} = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2} = \sqrt{\frac{1+1}{10^2}}$$

$$k = \frac{\sqrt{2}}{10}$$

$\theta = 0°$ (Since its in same phase)

Recall $\frac{x}{\sqrt{2}} = k\sin(t + \alpha)$

$$\frac{x}{\sqrt{2}} = \sqrt{2}/10 \sin 45 + \alpha)$$

$$1 = \sin(45 + \alpha)$$

$$45 + \alpha = \sin^{-1}(1)$$

$$\alpha = 90 - 45 = 45° = \frac{\pi}{4}$$

The Steady State Soln is $x = \frac{\sqrt{2}}{10}\sin\left(t + \frac{\pi}{4}\right)$