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16/ENG 06/063 MECHANICAL ENGINEERING

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Mathematics Assignment

ENG 381

1. The dynamic model of a body in motion performing damped forced vibrations is as on Equation (1),

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Given that when $t=0$, $x = 0.1$ and $dx/dt = 0$.

(i) Using the Auxiliary Equation Method, obtain the solution of the model in form of an expression having x as a function of t,

Soln

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

CF: $m^2 + 5m + 6 = 0$; $m^2 + 2m + 3m + 6 = 0$

$m(m+2) + 3(m+2) = 0$; $m_1 = -3$ & $m_2 = -2$

$\therefore\ x = Ae^{-3t} + Be^{-2t}$ //

PI: $f(t) = \cos t$; $x = C\cos t + D\sin t$

$dx/dt = -C\sin t + D\cos t$

$d^2x/dt^2 = -C\cos t - D\sin t$

$(-C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t)$
$= \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t$

$+ 6C\cos t + 6D\sin t = \cos t$

$(5D - C + 6C)\cos t + (-5C - D + 6D)\sin t$

$= \cos t + 0$

Relating L·H·S & R·H·S

$5C + 5D = 1 \quad \dots (1)$

$-5C + 5D = 0 \quad \dots (2)$

$5D = 5C$

$D = C$

Substituting for D in (1)

$5C + 5C = 1$

$10C = 1$

$C = 1/10 = D$

PI $\Rightarrow$ $x = \frac{\cos t}{10} + \frac{\sin t}{10}$

General soln $\Rightarrow$

$x = Ae^{-3t} + Be^{-2t} + \frac{\cos t}{10} + \frac{\sin t}{10}$

$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}(\cos t + \sin t)$

when $t=0$ & $x = 0.1$

$0.1 = A + B + \frac{1}{10}(1 + 0)$

$A + B = 0$ ... *

$$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} + \frac{1}{10}(-\sin t + \cos t)$$

when $dx/dt = 0$; $t = 0$

$$0 = -3A - 2B + 0.1(-0 + 1)$$

$$3A + 2B = 0.1 \quad \ldots **$$

from eqn *

$$A = -B$$

substituting for A in ** eqn

$$3(-B) + 2B = 0.1$$

$$-B = 0.1$$

$$B = -0.1$$

$$A = -B$$

$$A = 0.1$$

$$\therefore x = 0.1 * e^{-3t} - 0.1 * e^{-2t} + 0.1(\cos t + \sin t)$$

$$x = 0.1(e^{-3t} - e^{-2t} + \cos t + \sin t)$$

$$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \cos t + \sin t) \quad //$$

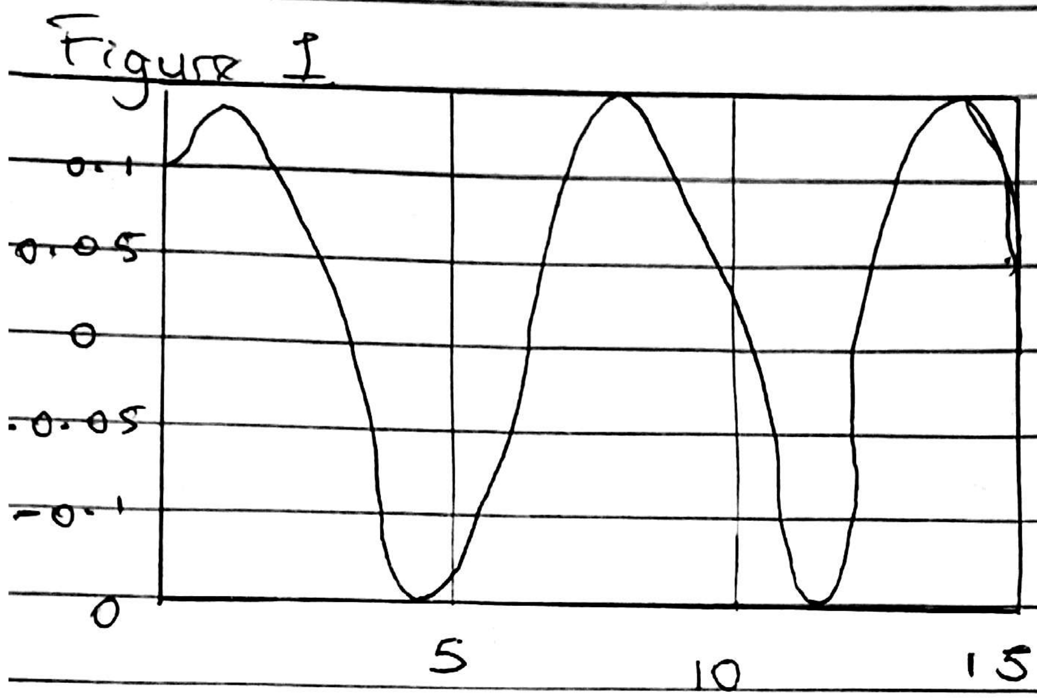
ii write a MATLAB program to plot the relationship between x & t for 0 ≤ t ≤ 15 unit using a step size of 0.01 unit

soln

MATLAB CODES

- commandwindow
- clear
- clc
- close all
- syms t
- x = 0.1 * (exp (-3*t) - exp (-2*t) + cos (t) + sin(t))
- tn = [0;0.01;15]
- xn = subs (x, tn)
- figure (1)
- Plot (tn , xn)
- grid on
- grid minor
- axis tight
- xlabel ~~('t')~~ ('time)
- ylabel ~~('x')~~ ('Vibrations')
- axis tight

Figure I



write the steadystate solution ~~fo~~ of the system in form of $x = k\sin(t + a)$

<u>soln</u>

$$x = \frac{1}{10}\left(e^{-3t} - e^{-2t} + \sin t + \cos t\right)$$

at steady state $\frac{dx}{dt} = 0$ i.e

change in x with time is zero

$$\therefore \frac{dx}{dt} = \frac{1}{10}\left(-3e^{-3t} - e^{-2t} \cancel{+ s} + \cos t - \sin t\right)$$

nb: the exponentials results zero

$$0 = \cos t - \sin t$$

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$\cos t = \sin t$

$t = 45^\circ$

$\therefore \; x = \frac{1}{10}(\cos 45 + \sin 45) = \sqrt{2}/10$

from sinosoodial expression

$A\cos\omega t + B\sin\omega t = K\cos(\omega t - \theta)$

But; $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where

$K = \sqrt{A^2 + B^2} = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2} = \sqrt{\frac{1+1}{(10)^2}}$

$K = \sqrt{2}/10$ //

$\theta = 0^\circ$ (since its in same phase

Recall $\quad x = K\sin(t + a)$

$\sqrt{2}/10 = \sqrt{2}/10 \sin(45 + a)$

$1 = \sin \cancel{\cos a} (45 + a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45 = 45^\circ$ or $\pi/4$

$\therefore$ The steady state soln is

$$x = \frac{\sqrt{2}}{10}\sin\left(t + \pi/4\right) //$$