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Chemical.

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t.$$

Let $x = e^{kt}$

$$\frac{dx}{dt} = ke^{kt} = kx, \quad \frac{d^2x}{dt^2} = k^2e^{kt} = k^2x.$$

$$k^2x + 5kx + 6x = 0.$$

$$x(k^2 + 5k + 6) = 0.$$

divide all through by x.

$$k^2 + 5k + 6 = 0 \quad \text{- Auxillary eqn.}$$

$$k = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$k = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$k_1 = \frac{-5 + \sqrt{1}}{2} \qquad k_2 = \frac{-5 - \sqrt{1}}{2}$$

$$k_1 = -2 \qquad k_2 = -3$$

$$x = Ae^{-2t} + Be^{-3t} \quad \text{- C.F.}$$

General form of the RHS. $\cos t$.

$$x = C\cos t + D\sin t.$$

$$\frac{dx}{dt} = -C\sin t + D\cos t$$

$$(-(C\cos t - D\sin t) + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t).$$

$$(-C\cos t - D\sin t) - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t.$$

$$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t$$

$$(5C + 5D)\cos t + (5D - 5C)\sin t = \cos t.$$

$$5C + 5D = 1$$

$$-5C + 5D = 0.$$

$$10C = 1$$

$$C = \frac{1}{10} = 0.1$$

$$5\left(\frac{1}{2}\right) + 5D = 1$$

$$\frac{1}{2} + 5D = 1$$

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$5D = 1 - \frac{1}{2}$

$5D = \frac{1}{2}$

$D = \frac{1}{2} \times \frac{1}{5} = \frac{1}{10} = 0.1$

Complete general solution

$x = Ae^{-2t} + Be^{-3t} + 0.1\cos t + 0.1\sin t$

$x = Ae^{-2t} + Be^{-3t} + 0.1(\cos t + \sin t)$

When $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$.

$0.1 = A + B + \frac{1}{10}$

$0.1 - \frac{1}{10} = A + B$

$0 = A + B$ --- (1)

When $\frac{dx}{dt} = 0$.

$0 = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$.

When $t = 0$.

$0 = -2A - 3B + \frac{1}{10} = -2A - 3B + 0.1$

$-0.1 = -2A - 3B$

$-0.1 = -2A - 3B$

$0 = 2A + 2B$

$-0.1 = +B$

$B = 0.1$ Sub into eqn (1).

$0 = A + 0.1$

$A = -0.1$

$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$

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iii $x_c = K\sin(t + a)$

To solve the steady state equation.

$$x = \frac{1}{10}(e^{-2t} + e^{-3t}) + (\cos t + \sin t)$$

Transient part of equation — Steady state part of equation.

Using the steady state part of the equation

$$x = 0.1\cos t + 0.1\sin t$$

note $\frac{dx}{dt} = 0$ for steady state.

$$\frac{dx}{dt} = 0.1(-\sin t + \cos t) = 0$$

$$= -\sin t + \cos t = 0$$

$$\cos t = \sin t$$

Hence: $t = 45^\circ$ or $\frac{\pi}{4}$

$$x = 0.1(\cos 45 + \sin 45)$$

Sinusodal equation:

$$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$$

But

$$\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$$

where

$$k = \sqrt{A^2 + B^2} = \sqrt{(0.1)^2 + (0.1)^2} = 0.14.$$

Since it is on the same phase, hence. $\theta = 0^\circ$

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