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Mechanical Engineering.
300L
16/ENG06/001
ENG 381

1.) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

C.f : $m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+3)(m+2) = 0$

$m_1 = -3$ and $m_2 = -2$

$\therefore Ae^{-3t} + Be^{-2t} = C.f$

P.I : $f(t) = \cos t$

$x = A\cos t + B\sin t$

$\frac{dx}{dt} = -A\sin t + B\cos t$

$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$

$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$[-A\cos t - B\sin t] + [-5A\sin t + 5B\cos t] + [6A\cos t + 6B\sin t] = \cos t$

$[-A\cos t + 6A\cos t - B\sin t + 6B\sin t - 5A\sin t + 5B\cos t] = \cos t$

$5A\cos t + 5B\sin t - 5A\sin t + 5B\cos t = \cos t$

Collecting the coefficients of like terms.

$5A + 5B = 1$

$+ \quad -5A + 5B = 0$

$10B = 1 \qquad B = 1/10$

$\therefore\ 5A + 5B = 1$

$5A + B\left(\frac{1}{10}\right) = 1$

$5A + \frac{1}{2} = 1$

$5A = 1 - \frac{1}{2} = \frac{1}{2}$

$10A = 1$

$A = \frac{1}{10}$

$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$P.I = \frac{1}{10}[\cos t + \sin t]$

$x = C.f + P.I$

$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}[\cos t + \sin t]$

$x = \left(\frac{1}{10}\right)^{-3t} + \left(\frac{1}{10}\right)^{-2t} + \frac{1}{10}(\cos t + \sin t)$

1.) Write a mat lab program to plot the relationship between x if for $0 \leq t \leq 15$ unit using a step size of 0.01 unit.

Soln

Command window

Clear

Clc

Close all

Syms t

where

$k = \sqrt{A^2 + B^2}$

$= \sqrt{(1/10)^2 + (1/10)^2} = \sqrt{\frac{1+1}{10^2}}$

$k = \sqrt{2}/10,$

$\theta = 0^\circ$ (the same phase).

Recall $x = k \sin(t + a)$

$\sqrt{2}/10 = \sqrt{2}/10 \sin(45 + a)$

$1 = \sin(45 + a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45 = 45^\circ \simeq \pi/4$

The steady state solution is, $x - \frac{\sqrt{2}}{10} \sin\left(t + \frac{\pi}{4}\right)$