

# Assignment 1

ADANABA CITIBANK

MECHANICAL ENGINEERING

16/06/2012

ENG 381

$$\textcircled{1} \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$CF = m^2 + 5m + 6 = 0$$

$$m^2 + 3m + 2m + 6 = 0$$

$$m(m+3) + 2(m+3) = 0$$

$$(m+3)(m+2) = 0$$

$$m_1 = -3 \text{ and } m_2 = -2$$

$$\therefore x = Ae^{-3t} + Be^{-2t} \quad \text{C.F.}$$

$$PI: f(t) = \cos t$$

$$x = A\cos t + B\sin t$$

$$\frac{dx}{dt} = -A\sin t + B\cos t$$

$$\frac{d^2x}{dt^2} = -A\cos t - B\sin t$$

$$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x$$

$$(-A\cos t - B\sin t) + (-5A\sin t + 5B\cos t) + (6A\cos t + 6B\sin t) = \cos t$$

$$(-A\cos t + 6A\cos t - B\sin t + 6B\sin t - 5A\sin t + 5B\cos t) = \cos t$$

$$= 5A\cos t + 5B\sin t - 5A\sin t + 5B\cos t = \cos t$$

Collecting the coefficient of like terms

$$5A + 5B = 1$$

$$-5A + 5B = 0$$

$$10B = 1$$

$$B = \frac{1}{10}$$

$$5A + 5B = 1$$

$$5A + 5\left(\frac{1}{10}\right) = 1$$

$$5A + \frac{1}{2} = 1$$

$$5A = 1 - \frac{1}{2}$$

$$10A = 1$$

$$A = \frac{1}{10}$$

$$P.I = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$P.I = \frac{1}{10} (\cos t + \sin t)$$

$$x = CF + P.I$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10} [\cos t + \sin t]$$

Write a matlab program to plot the relationship between  $x$  it for  $0 \leq t \leq 15$  unit using a step size of 0.01 unit

Soln

- Command Window
- Clear
- Clc
- Close all
- Symst

$$x = 0.1 * (\exp(-3*t) - \exp(-2*t)) + \cos(t) + \sin(t)$$

$$tn = [0, 0.01, 15]$$

$$x_n = \text{Subs}(x, t_n)$$

figure c1

Plot(tn, xn)

grid on

grid minor

axis tight

x label ('time')

y label ('vibrations')

11) Write the steady state solution of the system in form of  $x = k \sin(\omega t + \phi)$

$$x = \frac{1}{10} (e^{-3t} - e^{-2t} + \sin t + \cos t)$$

$$\text{at steady state } \frac{dx}{dt} = 0$$

Change in  $x$  with time is zero

$$\therefore \frac{dx}{dt} = \frac{1}{10} (-3e^{-3t} - e^{-2t} + \cos t - \sin t)$$

Note below: the exponentials result zero

$$0 = \cos t - \sin t$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$x = \frac{1}{10} (\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$$

from sinusoidal expression

$$A \cos \omega t + B \sin \omega t = k \cos(\omega t - \theta)$$

$$\text{But: } \cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$$

where

$$k = \sqrt{A^2 + B^2} = \sqrt{(\frac{1}{10})^2 + (\frac{1}{10})^2}$$

$$k = \frac{\sqrt{2}}{10}$$

$$\theta = 0^\circ \text{ (since it's in the same phase)}$$

$$\text{Recall } x = k \sin(\omega t + \phi)$$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10} \sin(45 + 90)$$



$$1 = \sin(45 + a)$$

$$45 + a = \sin^{-1}(1)$$

$$a = 90 - 45$$

$$= 45^\circ \approx \pi/4$$

The steady state seen is

$$x = \frac{\sqrt{2}}{10} \sin(t + \pi/4)$$