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16/ENG05001
MECHATRONICS

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Soln

aux equ: $m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+2)(m+3) = 0$

$\therefore m_1 = -2 \;\&\; m_2 = -3$

using solution form $y = Ae^{m_1x} + Be^{m_2x}$

$\therefore x = Ae^{-2t} + Be^{-3t}$

$C \cdot F : x = Ae^{-2t} + Be^{-3t}$

$P \cdot I \quad x = C\cos t + D\sin t$

$\therefore \frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$\therefore [-C\cos t - D\sin t] + 5[-C\sin t + D\cos t] + 6[C\cos t + D\sin t] = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$-C\cos t + 5D\cos t + 6C\cos t - D\sin t - 5C\sin t + 6D\sin t = \cos t$

$(-C + 5D + 6C)\cos t + \sin t(-D - 5C + 6D) = \cos t$

$(5C + 5D)\cos t + \sin t(5D - 5C) = \cos t$

Comparing Coefficients

$\cos t : 5C + 5D = 1$

$\sin t : 5D - 5C = 0$

$5D = 5C$

$D = C$

$\therefore 5C + 5C = 1$

$C = \frac{1}{10} \quad \therefore D = \frac{1}{10}$

$\therefore \quad x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$P.I \quad x = \frac{1}{10}(\cos t + \sin t)$

$\therefore$ General Solution

$$x = P.I + C.F$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}(\cos t + \sin t)$$

when $t = 0 \quad x = 0.1 \quad \& \quad \frac{dx}{dt} = 0$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}(\cos 0 + \sin 0)$$

$$0.1 = A + B + \frac{1}{10}(1 + 0)$$

$$A + B = \cancel{0.1 \times \frac{10}{1}} \; 0.1 - 0.1$$

$$A + B = 0$$

$$\frac{dx}{dt} = -2Ae^{-2t} + -3Be^{-3t} + -\frac{1}{10}\sin t + \frac{1}{10}\cos t$$

at $t = 0 \quad \frac{dx}{dt} = 0$

$$\therefore \quad 0 = -2Ae^{0} + -3Be^{0} - 0 + \frac{1}{10}$$

$$-\frac{1}{10} = -2A - 3B$$

$$\frac{1}{10} = 2A + 3B$$

$$\frac{1}{10} = -2B + 3B = B$$

$$\therefore \quad B = \frac{1}{10}$$

$$A = -\frac{1}{10}$$

$\therefore \quad x = -\frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} (\cos t + \sin t)$

$\therefore \quad x = 0.1 \left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$

iii) Steady State of form $x = k \sin (t + a)$

Soln

$x = \frac{1}{10} \left(e^{-3t} - e^{-2t} + \cos t + \sin t\right)$

at steady state $\frac{dx}{dt} = 0$

& we are considering the steady state part of the equation

$\therefore \quad \frac{dx}{dt} = \frac{1}{10} (-\sin t + \cos t)$

$\therefore \quad 0 = \cos t - \sin t$

$\sin t = \cos t$

$t = 45°$

$\therefore \quad x = \frac{1}{10} (\cos 45° + \sin 45°)$

recall $A \cos 2\omega t + B \sin 2\omega t = k \cos (2\omega t - \theta)$

$\cos(2\omega t - \theta) = \sin (2\omega t - \theta + 90°)$

$k = \sqrt{A^2 + B^2} = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2} = \sqrt{\frac{2}{100}}$

$k = \frac{\sqrt{2}}{10}$

$\theta = 0°$ (since its in same phase)

Recall $x = k \sin(t + a)$

$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10} \sin (45 + a)$

$1 = \sin (45 + a)$

$$45 + a = \sin^{-1}(1)$$

$$45 + a = 90$$

$$a = 45^\circ$$

$$a = \pi/4$$

$$\therefore \quad x = \frac{\sqrt{2}}{10} \sin(45^\circ + \pi/4)$$

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