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Mechanical Engr.

ENG 381

Assignment.

(1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$x = 1 \quad k^2 + 5k + 6 = 0$

$k^2 + 3k + 2k + 6 = 0$

$k(k+3) + 2(k+3) = 0$

$k = -2, -3.$

$y = Ae^{k_1 x} + Be^{k_2 x}$

$x = Ae^{-2t} + Be^{-3t}$

$x = C\cos t + D\sin t.$

$\frac{dx}{dt} = x' = -C\sin t + D\cos t.$

$\frac{d^2x}{dt^2} \Rightarrow x'' = -C\cos t - D\sin t.$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t.$

$x = Ae^{-3t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t.$

given $t=0, \quad x = 0.1, \quad \frac{dx}{dt} = 0.$

$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$\therefore -2Ae^{0} - 3Be^{0} - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0 = 0.$

$= -2A - 3B + \frac{1}{10} = 0$

$-2A - 3B = -\frac{1}{10}$

$2A + 3B = \frac{1}{10}$ ——— eqn (1)

$0.1 = Ae^{0} + Be^{0} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$0.1 = A + B + \frac{1}{10}$

$A + B = 0.1 - \frac{1}{10}$

$A + B = 0$ ——— eqn (2)

$A = -B$

$\therefore -2B + 3B = \frac{1}{10}$

$B = \frac{1}{10} \quad \therefore A = -\frac{1}{10}.$

$x = +\frac{1}{10}e^{-2t} + \frac{1}{10}e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$\Rightarrow x = \frac{1}{10}\left[e^{-2t} + e^{-3t} + \cos t + \sin t\right)$

3) The Steady State Solution is

$X = C\cos t + D\sin t$

Assume $C = k\cos a$ and $D = k\sin a$

$x = (k\cos a)\sin t + (k\sin a)\cos t$.

$X = k[\sin t\cos a + \cos t\sin a]$

$x = k[\sin t\cos a - \cos t(-\sin a)]$

$x = k\left[\cos\left(\frac{\pi}{2} - t\right)\cos(-a) - \sin\left(\frac{\pi}{2} - t\right)\sin(-a)\right]$.

$X = k[\cos((\pi/2 - t) + (-a))]$

$x = k\left[\cos\frac{\pi}{2}\cos(t + a) + \sin\frac{\pi}{2}\sin(t + a)\right]$

$x = k[0 \times \cos(t + a) + 1 \times \sin(t + a)]$

$x = k[\sin(t + a)]$

$x = k\sin(t + a)$

Therefore, the Steady State Solution $X = C\cos t + D\sin t$ can be written on the form

$x = k\sin(t + a)$.