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16/EEN004 1034
Electrical Electronics
300 level

Solution

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

Auxiliary equation = $m^2 + 5m + 6$
 $(m+2)(m+3)$

$$m_1 = -2, m_2 = -3$$

C.F: $y = Ae^{-2x} + Be^{-3x}$

P.T $\Rightarrow y = C \cos t + D \sin t$

$$\frac{dy}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2y}{dt^2} = -C \cos t - D \sin t$$

Sub $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ into original eqn.

$$[-C \cos t - D \sin t] + 5[-C \sin t + D \cos t] + 6[C \cos t + D \sin t] = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$\cos t [-C + 5D + 6C] + \sin t [-D - 5C + 6D] = \cos t$$

$$\cos t [5D + 5C] - \sin t [-5C + 5D] = \cos t$$

Comparing coefficient.

$$\text{Cost: } 5D + 5C = 1$$

$$\text{Sint: } -5C + 6D = 0$$

$$\text{Cost: } 5D + 5C = 1 \quad \text{--- (1)}$$

$$\text{Sint: } 5D - 5C = 0 \quad \text{--- (2)}$$

Solving simultaneously.

$$5D + 5C = 1$$

$$5D - 5C = 0$$

$$10D - 0 = 1$$

$$10D = 1$$

$$D = \frac{1}{10}$$

input $D = \frac{1}{10}$ in eqn (2)

$$5D - 5C = 0$$

$$5\left(\frac{1}{10}\right) - 5C = 0$$

$$\frac{1}{2} - 5C = 0$$

$$5C = \frac{1}{2}$$

$$C = \frac{1}{2} \times \frac{1}{5}$$

$$C = \frac{1}{10}$$

$$C = \frac{1}{10}, \quad D = \frac{1}{10} \quad \therefore \text{P.I} = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$G.S = C.F + P.I$$

$$y = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

(ii) $x = K \sin(t + a)$
 $\therefore x = \frac{1}{10} [C_1 e^{-2t} + e^{-3t}] + (\cos t + \sin t)$

using steady part of the equation.
 $x = \frac{1}{10} (\cos t + \sin t)$

Note: $\frac{dx}{dt} = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10} (-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

Hence: $t = 45^\circ$ or $\pi/4$

$$x = \frac{1}{10} (\cos 45 + \sin 45)$$

Sinusoidal equation: $A \cos \omega t + B \sin \omega t = K \cos(\omega t - \theta)$

but $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90^\circ)$

where: $K = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$

Since it is on the same phase, hence: $\theta = 0^\circ$

$$x = \frac{1}{10} (\cos 45 + \sin 45)$$

$$= 0.14 \sin(45 + 90^\circ)$$

$$x = 0.14 \sin(45 + 90^\circ)$$

$$x = 0.14 \sin(90 + 45)$$

in terms of the above equation, hence

$$K = 0.14 \text{ or } \frac{\sqrt{2}}{10}$$

$$a = 45 \text{ or } \frac{\pi}{2}$$

$$x = 0.14 \sin(t + 45)$$

using the values of $t=0$, $x=0.1$, $\frac{dx}{dt} = 0$

$$y = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$0.1 = A + B + \frac{1}{10} + 0$$

$$0.1 = A + B + \frac{1}{10}$$

$$0.1 - \frac{1}{10} = A + B$$

$$A + B = 0$$

$$A = -B$$

$$\frac{dx}{dt} = 2Ae^{-2t} + 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$\therefore \frac{dx}{dt} = -2Be^{-2t} + 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -2B + 3B - 0 + \frac{1}{10}$$

$$0 = B + \frac{1}{10}$$

$$B = -0.1$$

$$A = 0.1$$

$$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$y = 0.1e^{-2t} - 0.1e^{-3t} + \frac{1}{10} (\cos t + \sin t)$$

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x =
exp(-3*t)/10 - exp(-2*t) + cos(t) + sin(t)

tn =
    0
    0.0100
    15.0000

xn =
cos(1/100) - exp(-1/50) + exp(-3/100)/10 + sin(1/100)
cos(15) - exp(-30) + exp(-45)/10 + sin(15)

>>

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COPMMANDS

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commandwindow
clear
clc
close all
syms t
x = 0.1*(exp(-3*t))-exp(-2*t)+cos(t)+sin(t)
tn = [0;0.01;15]
xn = subs(x,tn)
figure (1)
plot(tn,xn)
grid on
grid minor
axis tight
xlabel ('t')
ylabel ('x')

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