ENG 361 ASSIGNMENT

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1a) $\frac{d^3x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

Auxiliary Equation

$m^2 + 5m + 6m = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-5 \pm \sqrt{25 - 24}}{2}$

$m = \frac{-5 \pm 1}{2}$

$m = -2$ or 3

C.f $\rightarrow x = Ae^{-2t} + Be^{-3t}$

P.I

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(-C + 5D + 6C)\cos t + (-D - 5C + 6D)\sin t = \cos t$

Equating coefficient

$-C + 5D + 6C = 1$

$5C + 5D = 1$ — ①

$-D - 5C + 6D = 0$

$-5C + 5D = 0$ — ②

$5C + 5D = 1$

$-5C + 5D = 0$

$10C = 1$

P.T.O →

$C = \frac{1}{10}$

$5\left(\frac{1}{10}\right) + 5D = 1$

$\frac{1}{2} + 5D = 1$

$5D = 1 - \frac{1}{2}$

$5D = \frac{1}{2}$

$D = \frac{1}{10}$

$x_c = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

General solution = C.F + P.I

$= Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

3. Steady State Equation

$x = \frac{1}{10}\left(-e^{-2t} + e^{3t}\right) + (\cos t + \sin t)$

The transient part of the equation will not be considered since we are dealing with the steady state

$\therefore x = \frac{1}{10}(\cos t + \sin t)$

we know $\frac{dx}{dt} = 0$ for steady state

$\therefore \frac{dx}{dt} = \frac{1}{10}(\sin t + \cos t) \cdot 0$

$\therefore -\sin t + \cos t = 0$

$\therefore \cos t = \sin t$

$\therefore t = 45°$

$\therefore x = \frac{1}{10}(\cos 45 + \sin 45)$

$x = \frac{\sqrt{2}}{10}$

From Sinusoidal Expression

$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$

But $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90)$

where $k = \sqrt{A^2 + B^2} = \sqrt{\left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^2} = \frac{\sqrt{1+1}}{10}$

$k \frac{\sqrt{2}}{10}$

$\phi = 0°$ (since it is in same phase)

Recall $x = A\sin(t+\phi)$

$$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45+\phi)$$

$$\phi = 90-45 = 45° = \frac{\pi}{4}$$

steady state solution:

$$x = \frac{\sqrt{2}}{10}\left(\sin t + \frac{\pi}{4}\right)$$

since