

ENG381

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16/ENG05/021

Mechanics

$$\textcircled{1} \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$f(x) = 0$$

~~$$m^2 + 5m + 6 = 0$$~~

~~$$m = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 6}}{2} = \frac{-5 \pm \sqrt{25 - 24}}{2} = \frac{-5 \pm 1}{2}$$~~

$$m = \frac{-5+1}{2} \text{ or } \frac{-5-1}{2}$$

$$m = -2 \text{ or } -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

Particular Integral

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

Substituting

$$[-C \cos t - D \sin t] + 5[-C \sin t + D \cos t] + 6[C \cos t + D \sin t] = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 6C \cos t + 5D \cos t - D \sin t + 6D \sin t - 5C \sin t = \cos t$$

$$(5C + 5D) \cos t + (5D - 5C) \sin t = \cos t$$

$$5D + 5C = 1$$

$$5D - 5C = 0$$

$$10D = 1$$

$$D = \frac{1}{10}$$

$$5D + 5C = 1$$

$$5\left[\frac{1}{10}\right] + 5C = 1$$

$$5c = 0.5$$

$$c = 0.5/5 = 1/10$$

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

When $t=0$; $x=0.1$, $dx/dt = 0$

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0$$

$$dx/dt = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$0 = -2A - 3B + \frac{1}{10}$$

$$2A + 3B = \frac{1}{10}$$

$$A = -B$$

$$-2B + 3B = \frac{1}{10}$$

$$B = \frac{1}{10}$$

$$A = -\frac{1}{10}$$

$$x = -\frac{1}{10} e^{-2t} + \frac{1}{10} e^{-3t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

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Command window

clear

clc

close all

Syms t

$$x(t) = 0.1 * ((\exp(-3*t)) - (\exp(-2*t))) + \cos(t) + \sin(t)$$

$$tn = [0:0.01:15]$$

$$xd = \text{subs}(x(t), tn)$$

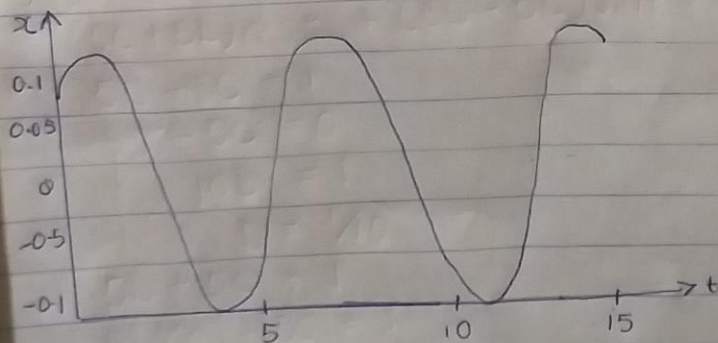
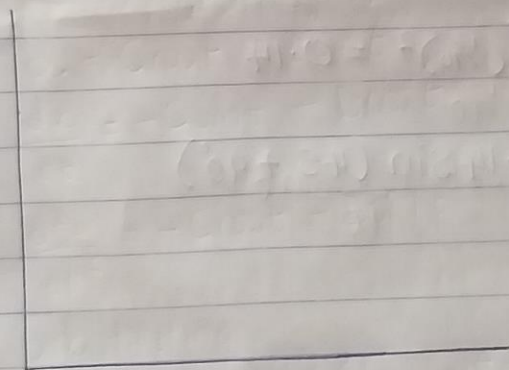
Figure (1)

Plot(tn, xd)

grid minor

grid on

axis



(iii) The steady state is $x = C \cos t + D \sin t$

$$x = \frac{1}{10} (-e^{-2t} + e^{-3t}) + (\cos t + \sin t)$$

$$x = \frac{1}{10} (\cos t + \sin t)$$

$$\frac{dx}{dt} = \frac{1}{10} (-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

$$t = 45^\circ$$

$$x = \frac{1}{10} (\cos 45 + \sin 45)$$

Recall: $A \cos \omega t + B \sin \omega t = K \cos(\omega t - \theta)$

$$\cos(\omega t - \theta) = \sin(\omega t - 0 + 90^\circ)$$

where

$$K = \sqrt{A^2 + B^2} = \sqrt{(\frac{1}{10})^2 + (\frac{1}{10})^2} = 0.14$$

$$x = \frac{1}{10} (\cos 45 + \sin 45) = 0.14 \sin(45 + 90^\circ)$$

$$x = 0.14 \sin(45 + 90^\circ)$$