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16/EN 606/061

MECHANICAL ENG

300L ENG 381

$$i) \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$m^2 + 5m + 6 = 0$$

$$m^2 + 2m + 3m + 6 = 0$$

$$(m^2 + 2m) + (3m + 6) = 0$$

$$m(m+2) + 3(m+2) = 0$$

$$\therefore m = -2, -3$$

$$\text{from } x = Ae^{m_1 t} + Be^{m_2 t}$$

$$x = Ae^{-2t} + Be^{-3t}$$

$$\text{For P.I, } x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$(-C \cos t - D \sin t) + 5(-C \sin t + D \cos t)$$

$$+ 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t$$

$$+ 6D \sin t = \cos t$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t = \cos t$$

$$\Rightarrow C + 5D + 6C = 1, \quad -D - 5C + 6D = 0$$

$$5C + 5D = 1, \quad 5D - 5C = 0$$

$$5C + 5D = 1$$

$$-5C + 5D = 0$$

$$10C = 1, \quad C = \frac{1}{10}$$

$$5(\frac{1}{10}) + 5D = 1$$

$$\frac{5}{10} - 1 = -5D$$

$$\therefore D = +\frac{1}{10}, \quad \therefore x = \frac{1}{10} \cos t - \frac{1}{10} \sin t$$

$$x = \frac{1}{10} (\cos t + \sin t)$$

$$\therefore x = Ae^{-2t} + Be^{-3t} + \frac{1}{10} (\cos t + \sin t) \dots \text{eqn 1}$$

$$\therefore \frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} + \frac{1}{10} (-\sin t + \cos t) \dots \text{eqn 2}$$

from eqn 1

$$\text{given } t=0, x=0.1 \quad \frac{dx}{dt} = 0$$

$$0.1 = A + B + 0.1(1) \dots$$

$$\therefore A + B = 0 \dots \text{eqn 3}$$

from eqn 2

$$-2A - 3B + 0.1(-1) = 0$$

$$-2A - 3B = 0.1 \dots \text{eqn 4}$$

$$A = -B$$

$$-2(-B) - 3B = 0.1$$

$$+2B - 3B = 0.1$$

$$-B = 0.1 \therefore B = -0.1$$

$$A = 0.1$$

$$\therefore x = 0.1 \times e^{-2t} - 0.1 \times e^{-3t}$$

$$+ 0.1 (\cos t + \sin t)$$

$$x = 0.1 (e^{-2t} - e^{-3t} + \cos t + \sin t)$$

$$\therefore x = \frac{1}{10} (e^{-2t} - e^{-3t} + \cos t + \sin t)$$

ii) MATLAB program to plot the relationship between x and t for $0 \leq t \leq 15$ unit using a step size of 0.01 unit

SSh

- Command window
- clear
- clc
- close all
- symst
- $x = 0.1 * (\exp(-3*t) - \exp(-2*t)) + \cos(t) + \sin(t)$
- $tn = [0; 0.01; 15]$
- $xn = \text{subs}(x, tn)$
- figure (1)
- plot (tn, xn)
- grid on
- grid mirror
- ~~grid~~ tight
- x label ('t')
- y label ('x')

Figure 1

