ENGINEERING MATHEMATICS ASSIGNMENT I

PROBLEM AND SOLUTION

The dynamic model of a body in motion performing damped forced vibrations is as in Equation (1),

$d^2x/dt^2 + 5dx/dt + 6x = \cos t$ …………………………………………………………….. (1)

Given that when t=0, x=0.1 and dx/dt = 0,

1) Using the Auxiliary Equation Method, obtain the solution of the model in the form of an expression having x as a function of t.

SOLUTION

For the complementary function (CF),

$d^2x/dt^2 + 5dx/dt + 6x = \cos t$ …………(1)

Assume

$f(t) = 0$ ……………………………..(i)

$x = Ae^{kt}$ ………………………………………….(ii)

$dx/dt = kAe^{kt} = kx$ …………………………….(iii)

$d^2x/dt^2 = k^2Ae^{kt} = k^2x$ ………………………..(iv)

By substituting equations (i), (ii), (iii) and (iv) into equation (1), equation (1) becomes:

$k^2x + 5kx + 6x = 0$

$x(k^2 + 5k + 6) = 0$

$x = 0$ and

$k^2 + 5k + 6 = 0$ …………………………..Auxiliary Equation

Using factorization method,

$k^2 + 2k + 3k + 6 = 0$

$k(k+2) + 3(k+2) = 0$

$(k+2)(k+3) = 0$

$K = -2$ or -3

Hence;

$K_1 = -2$ and $K_2 = -3$

Thus, the complementary function of equation (1) becomes:

$x = Ae^{-2t} + Be^{-3t}$ ………………………………………. (2)

+For the particular integral (PI),

Assume

$x = C\sin t + D\cos t$ …………………… (I)

$dx/dt = C\cos t - D\sin t$ ………………. (II)

$d^2x/dt^2 = -C\sin t - D\cos t = -(C\sin t + D\cos t)$ ……………(III)

By substituting equations (I), (II) and (III) into equation (1), equation (1) becomes:

$-C\sin t - D\cos t + 5(C\cos t - D\sin t) + 6(C\sin t + D\cos t) = \cos t$

$(-D+5C + 6D)\cos t + (-C - 5D + 6C)\sin t = \cos t$

By comparing coefficients;

cost: $5C + 5D = 1$ ………………..(IV)

sint: $5C - 5D = 0$ ………………….(V)

Equation (IV) + equation (V)

$10 C = 1, C = 1$

Put $C = 1/10$ into equation (IV),

$5(1/10) + 5D = 1;$

$5D = 1 - (1/2) = 1/2$

$D = (1/2) X (1/5) = 1/10$

Putting the values of C and D into equation (I), equation (I) which is the (PI) becomes:

$x = (1/10)\sin t + (1/10)\cos t$

$x = (1/10)(\sin t + \cos t)$ ……………………………………………. (3)

Hence:

The complete solution of the equation using the auxiliary equation method is:

$x = Ae^{-2t} + Be^{-3t} + (1/10)(\sin t + \cos t)$ …………………………… (4)

By putting $x = 0.1$ and $t = 0$ into equation (4), equation (3) becomes:

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + (1/10)(\sin(0) + \cos(0))$
$0.1 = A + B + 1/10$

$A + B = 1/10 - 1/10 = 0;$

$A + B = 0, A = -B$

Also, by differentiating x:

$dx/dt = -2Ae^{-2t} -3Be^{-3t} + (1/10)\ (\cos t - \sin t)$

Putting $dx/dt = 0$, $t = 0$ and $A = -B$

$0 = -2(-B)e^{-2(0)} -3Be^{-3(0)} + (1/10)\ (\cos(0) – \sin(0))$

$0 = 2B -3B + 1/10$

$B = 1/10$

$A = -B = - (1/10)$

Hence the complete particular solution becomes:

$x = (1/10)(-e^{-2t} + e^{-3t}) + (1/10)\ (\sin t +\cos t)$

$x = (1/10)(-e^{-2t} + e^{-3t} + \sin t +\cos t)$

2) Write a MATLAB program to plot the relationship between x and t for 0 <= t<= 15 unit using a step size of 0.01 unit.

Solution

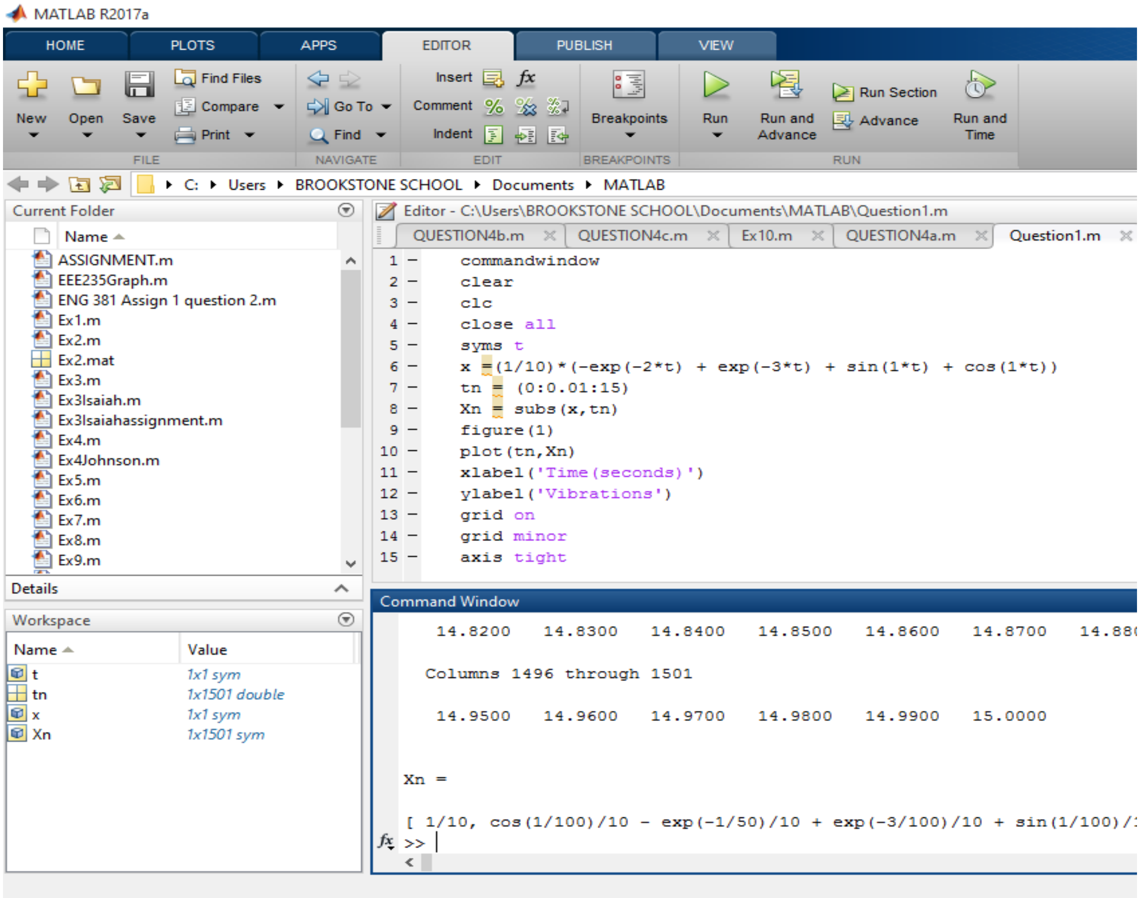

Figure 1.1: Code from Mathlab for plotting the graph of x against t.

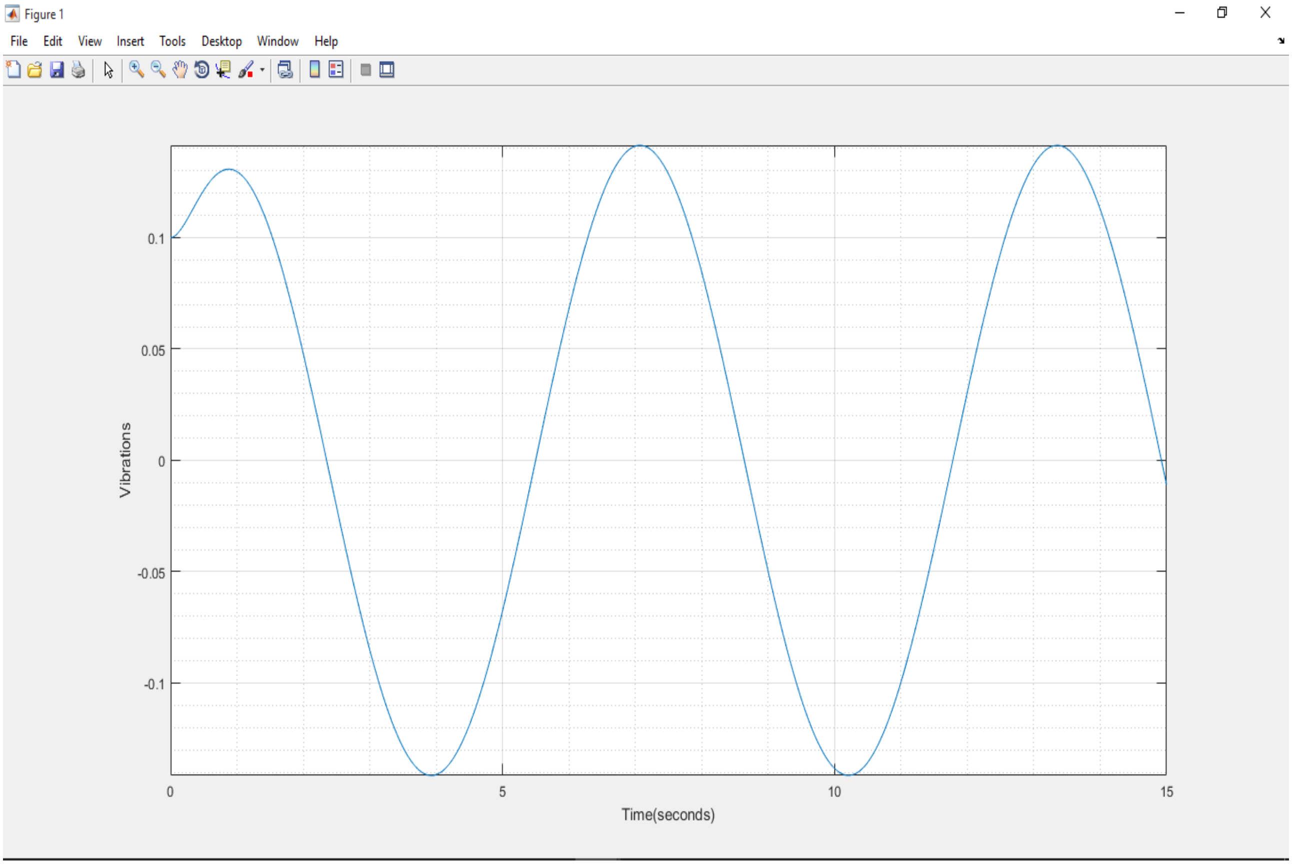

Figure 1.2: graph showing relationship between x and t from 0<=t<=15 unit.

3) Write the steady-state solution of the system in the form of x = K sin (t + a).

SOLUTION

The solution of the equation contains both transient part and a steady state part which are as follows:

Transient part = 1/10(e-2t + e-3t)

Steady state part = 1/10 (sint + cost) = 1/10 sin t + 1/10 cost

Using trigonometric method of adding trigonometric functions,

Asint + B cost = C sin (t + θ)

Where θ = tan-1(A/B)

$C = (A^2 + B^2)^{1/2}$

Using that in our steady state part, the steady part becomes:

1/10sint + 1/10cost = Ksin(t+a)

Where;

$K = ((-0.1)^2 + 0.1^2)^{1/2} = (2)^{1/2}/10$

a = tan-1(-0.1/0.1) = -45◦

Hence the steady state solution of the system in the form x = Ksin (t+a) is:

$X = ((2)^{1/2}/10)(\sin(t-45^\circ))$