

**ENGINEERING MATHEMATICS ASSIGNMENT I**

**PROBLEM AND SOLUTION**

The dynamic model of a body in motion performing damped forced vibrations is as in Equation (1),

$$d^2x/dt^2 + 5dx/dt + 6x = \cos t \dots\dots\dots (1)$$

Given that when  $t=0$ ,  $x=0.1$  and  $dx/dt = 0$ ,

- 1) Using the Auxiliary Equation Method, obtain the solution of the model in the form of an expression having  $x$  as a function of  $t$ .

**SOLUTION**

For the complementary function (CF),

$$d^2x/dt^2 + 5dx/dt + 6x = \cos t \dots\dots\dots(1)$$

Assume

$$f(t) = 0 \dots\dots\dots(i)$$

$$x = Ae^{kt} \dots\dots\dots(ii)$$

$$dx/dt = kAe^{kt} = kx \dots\dots\dots(iii)$$

$$d^2x/dt^2 = k^2Ae^{kt} = k^2x \dots\dots\dots(iv)$$

By substituting equations (i), (ii), (iii) and (iv) into equation (1), equation (1) becomes:

$$k^2x + 5kx + 6x = 0$$

$$x(k^2 + 5k + 6) = 0$$

$$x = 0 \text{ and}$$

$$k^2 + 5k + 6 = 0 \dots\dots\dots\text{Auxiliary Equation}$$

Using factorization method,

$$k^2 + 2k + 3k + 6 = 0$$

$$k(k+2) + 3(k+2) = 0$$

$$(k+2)(k+3) = 0$$

$$K = -2 \text{ or } -3$$

Hence;

$$K_1 = -2 \text{ and } K_2 = -3$$

Thus, the complementary function of equation (1) becomes:

$$x = Ae^{-2t} + Be^{-3t} \dots\dots\dots (2)$$

+For the particular integral (PI),

Assume

$$x = C\sin t + D\cos t \dots\dots\dots (I)$$

$$dx/dt = C\cos t - D\sin t \dots\dots\dots (II)$$

$$d^2x/dt^2 = -C\sin t - D\cos t = - (C\sin t + D\cos t ) \dots\dots\dots(III)$$

By substituting equations (I), (II) and (III) into equation (1), equation (1) becomes:

$$-C\sin t - D\cos t + 5(C\cos t - D\sin t) + 6 (C\sin t + D\cos t ) = \cos t$$

$$(-D+5C + 6D)\cos t + (-C - 5D + 6C)\sin t = \cos t$$

By comparing coefficients;

$$\cos t: 5C + 5D = 1 \dots\dots\dots(IV)$$

$$\sin t: 5C - 5D = 0 \dots\dots\dots(V)$$

Equation (IV) + equation (V)

$$10 C = 1, C = 1/10$$

Put C = 1/10 into equation (IV),

$$5(1/10) + 5D = 1;$$

$$5D = 1 - (1/2) = 1/2$$

$$D = (1/2) \times (1/5) = 1/10$$

Putting the values of C and D into equation (I), equation (I) which is the (PI) becomes:

$$x = (1/10)\sin t + (1/10)\cos t$$

$$x = (1/10) (\sin t + \cos t) \dots\dots\dots (3)$$

Hence:

The complete solution of the equation using the auxiliary equation method is:

$$x = Ae^{-2t} + Be^{-3t} + (1/10) (\sin t + \cos t) \dots\dots\dots (4)$$

By putting x = 0.1 and t = 0 into equation (4), equation (3) becomes:

$$0.1 = Ae^{-2(0)} + Be^{-3(0)} + (1/10) (\sin(0) + \cos(0))$$

$$0.1 = A + B + 1/10$$

$$A + B = 1/10 - 1/10 = 0;$$

$$A + B = 0, A = - B$$

Also, by differentiating x:

$$dx/dt = -2Ae^{-2t} - 3Be^{-3t} + (1/10)(\cos t - \sin t)$$

Putting  $dx/dt = 0$ ,  $t = 0$  and  $A = -B$

$$0 = -2(-B)e^{-2(0)} - 3Be^{-3(0)} + (1/10)(\cos(0) - \sin(0))$$

$$0 = 2B - 3B + 1/10$$

$$B = 1/10$$

$$A = -B = -(1/10)$$

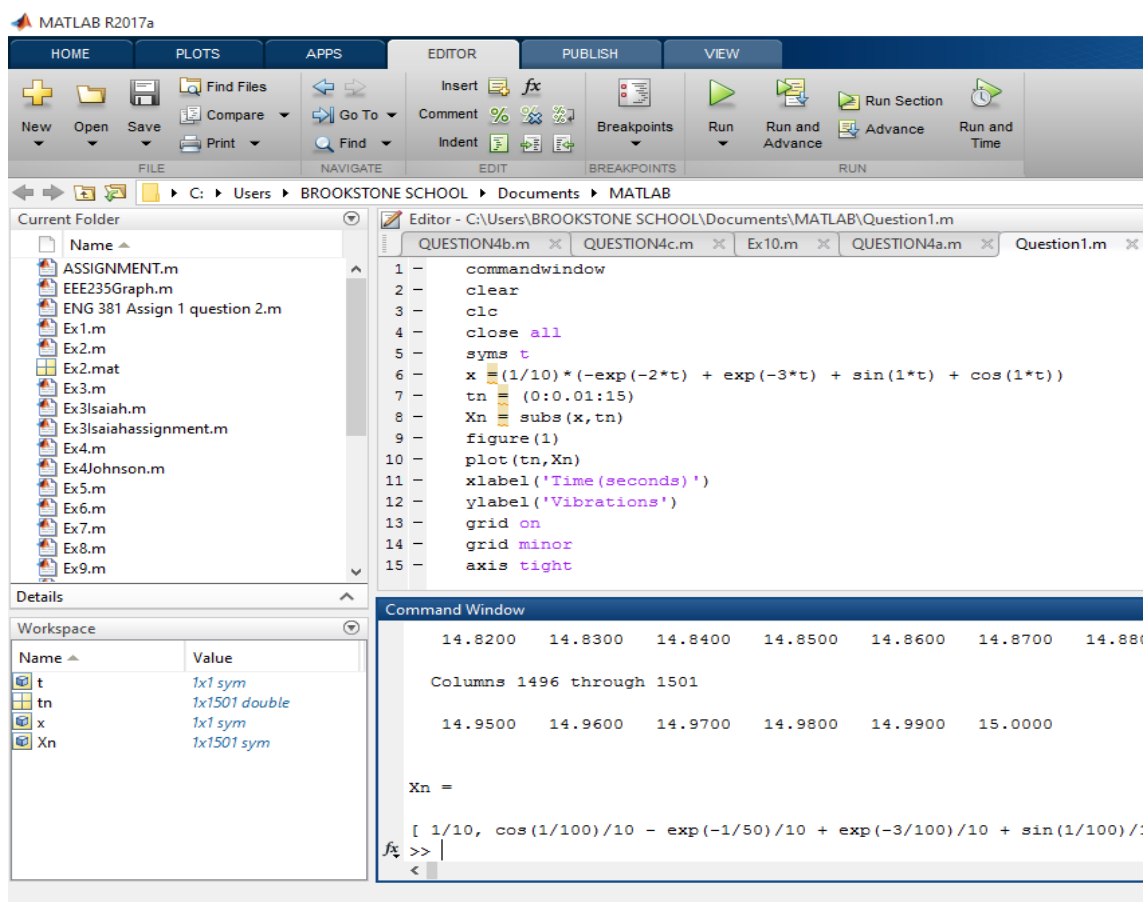
Hence the complete particular solution becomes:

$$x = (1/10)(-e^{-2t} + e^{-3t}) + (1/10)(\sin t + \cos t)$$

$$x = (1/10)(-e^{-2t} + e^{-3t} + \sin t + \cos t)$$

- 2) Write a MATLAB program to plot the relationship between  $x$  and  $t$  for  $0 \leq t \leq 15$  unit using a step size of 0.01 unit.

### Solution



MATLAB R2017a

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- Ex2.m
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- Ex3Isaiahassignment.m
- Ex4.m
- Ex4Johnson.m
- Ex5.m
- Ex6.m
- Ex7.m
- Ex8.m
- Ex9.m

Workspace

Name	Value
t	1x1 sym
tn	1x1501 double
x	1x1 sym
Xn	1x1501 sym

```

1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - syms t
6 - x = (1/10)*(-exp(-2*t) + exp(-3*t) + sin(1*t) + cos(1*t))
7 - tn = (0:0.01:15)
8 - Xn = subs(x,tn)
9 - figure(1)
10 - plot(tn,Xn)
11 - xlabel('Time (seconds)')
12 - ylabel('Vibrations')
13 - grid on
14 - grid minor
15 - axis tight

```

Command Window

```

14.8200 14.8300 14.8400 14.8500 14.8600 14.8700 14.8800
Columns 1496 through 1501
14.9500 14.9600 14.9700 14.9800 14.9900 15.0000
Xn =
[ 1/10, cos(1/100)/10 - exp(-1/50)/10 + exp(-3/100)/10 + sin(1/100)]:
fx >> |
<

```

Figure 1.1: Code from Matlab for plotting the graph of  $x$  against  $t$ .

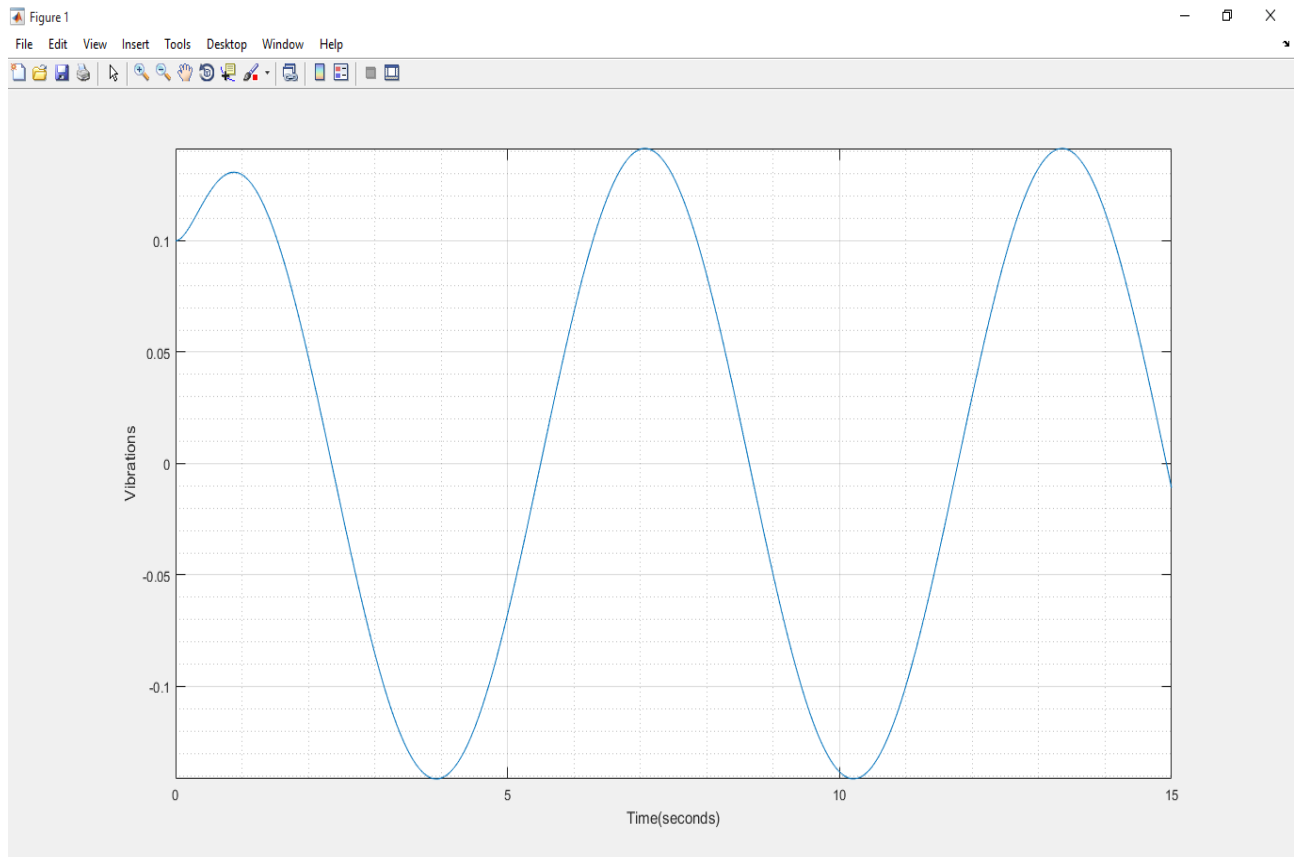


Figure 1.2: graph showing relationship between  $x$  and  $t$  from  $0 \leq t \leq 15$  unit.

- 3) Write the steady-state solution of the system in the form of  $x = K \sin(t + a)$ .

**SOLUTION**

The solution of the equation contains both transient part and a steady state part which are as follows:

Transient part =  $1/10(e^{-2t} + e^{-3t})$

Steady state part =  $1/10(\sin t + \cos t) = 1/10 \sin t + 1/10 \cos t$

Using trigonometric method of adding trigonometric functions,

$A \sin t + B \cos t = C \sin(t + \theta)$

Where  $\theta = \tan^{-1}(A/B)$

$C = (A^2 + B^2)^{1/2}$

Using that in our steady state part, the steady part becomes:

$1/10 \sin t + 1/10 \cos t = K \sin(t+a)$

Where;

$K = ((-0.1)^2 + 0.1^2)^{1/2} = (2)^{1/2}/10$

$a = \tan^{-1}(-0.1/0.1) = -45^\circ$

Hence the steady state solution of the system in the form  $x = K \sin(t+a)$  is:

$X = ((2)^{1/2}/10)(\sin(t-45^\circ))$

