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PETROLEUM ENG
16/ENG07/005
ENG381

1) $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

assuming $\cos t = 0$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m_1 = -2$ or $m_2 = -3$

C.F $\Rightarrow x = Ae^{-2t} + Be^{-3t}$

P.I $\Rightarrow x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\sin t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) = \cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$\cos t(-C + 5D + 6C) + \sin t(-D - 5C + 6D) = \cos t$

$\cos t(5C + 5D) + \sin t(5D - 5C) = \cos t$

$\cos t(5C + 5D) = \cos t$

$5C + 5D = 1 \quad \cdots (1)$

$\sin t(5D - 5C) = 0$

$5D - 5C = 0 \quad \cdots (2)$

Solve equ. (1) and (2) simultaneously

$10D = 1$

$D = \frac{1}{10}$

Substitute $\frac{1}{10}$ for D in equ. (1)

$5C + 5D = 1$

$5c + 5\left(\frac{1}{10}\right) = 1$

$5c + \frac{5}{10} = 1$

$5c + \frac{1}{2} = 1$

$5c = 1 - \frac{1}{2}$

$5c = \frac{1}{2}$

$c = \frac{1}{10}$

$P.I \Rightarrow x = \frac{1}{10}\cos t + \frac{1}{10}\sin t$

$G.S = C.F + P.I$

$G.S \Rightarrow x = Ae^{-2t} + Be^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

when $t = 0$, $x = 0.1$, $\frac{dx}{dt} = 0$

$0.1 = Ae^{-2(0)} + Be^{-3(0)} + \frac{1}{10}\cos 0 + \frac{1}{10}\sin 0$

$0.1 = A + B + \frac{1}{10}$

$A + B = 0.1 - \frac{1}{10}$

$A + B = 0$

$A = -B \quad ---(3)$

$\frac{dx}{dt} = -2Ae^{-2t} - 3Be^{-3t} - \frac{1}{10}\sin t + \frac{1}{10}\cos t$

$0 = -2Ae^{-2(0)} - 3Be^{-3(0)} - \frac{1}{10}\sin 0 + \frac{1}{10}\cos 0$

$0 = -2A - 3B + 0.1$

$2A + 3B = 0.1 \quad ---(4)$

Sub. equ.(3) into equ.(4)

$2(-B) + 3B = 0.1$

$-2B + 3B = 0.1$

$\therefore B = 0.1$

$\therefore A = -0.1$

P.S $\Rightarrow x = -0.1e^{-2t} + 0.1e^{-3t} + \frac{1}{10}\cos t + \frac{1}{10}\sin t$

ii) MATLAB program:

```
Command window
clear
clc
close all
syms t
x(t) = 0.1*((exp(-3*t)) - (exp(-2*t)) + cos(t) + sin(t))
tn = [0:0.01:15]
xd = subs(x(t), tn)
figure 1
plot(tn, xd)
axis tight
grid on
grid minor
```

iii) Steady state solution in form of $x = K\sin(t+a)$

~~From eq(2)~~ Using the P.I from the General solution:

~~Also~~ $x = C\cos t + D\sin t$

Assume $C = K\cos a$ and $D = K\sin a$

$x = (K\cos a)\sin t + (K\sin a)\cos t$

$x = K[\sin t\cos a + \cos t\sin a]$

$x = K[\sin t\cos a - \cos t(-\sin a)]$

$x = K[\cos(\frac{\pi}{2} - t)\cos(-a) - \sin(\frac{\pi}{2} - t)\sin(-a)]$

$x = K[\cos((\frac{\pi}{2} - t) + (-a))]$

$x = K[\cos((\frac{\pi}{2} - t) - a)]$

$x = K[\cos(\frac{\pi}{2} - (t+a)]$

$x = K[\cos\frac{\pi}{2}\cos(t+a) + \sin\frac{\pi}{2}\sin(t+a)]$

$x = K[0*\cos(t+a) + 1*\sin(t+a)]$

$x = K\sin(t+a)$