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Mechanical Engineering
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ENG-381

① $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$

$CF = m^2 + 5m + 6 = 0$

$m^2 + 3m + 2m + 6 = 0$

$m(m+3) + 2(m+3) = 0$

$(m+3)(m+2) = 0$

$m_1 = -3$ and $m_2 = -2$

$\therefore x = Ae^{-3t} + Be^{-2t} = CF$

PI: $f(t) = \cos t$

$x = A\cos t + B\sin t$

$dx/dt = -A\sin t + B\cos t$

$d^2x/dt^2 = -A\cos t - B\sin t$

$\therefore \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x$

$(-A\cos t - B\sin t) + (-5A\sin t + 5B\cos t) + (6A\cos t + 6B\sin t) = \cos t$

$(-A\cos t + 6A\cos t - B\sin t + 6B\sin t - 5A\sin t + 5B\cos t) = \cos t$

$= 5A\cos t + 5B\sin t - 5A\sin t + 5B\cos t = \cos t$

Collecting the coefficient of like terms

$5A + 5B = 1$

$-5A + 5B = 0$

$10B = 1$

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$$B = \frac{1}{10}$$

$$\therefore 5A + 5B = 1$$

$$5A + 5\left(\frac{1}{10}\right) = 1$$

$$5A + \frac{1}{2} = 1$$

$$5A = 1 - \frac{1}{2}$$

$$10A = 1$$

$$A = \frac{1}{10}$$

$$P.I = \frac{1}{10}\cos t + \frac{1}{10}\sin t$$

$$P.I = \frac{1}{10}(\cos t + \sin t)$$

$$x = CF + P.I$$

$$x = Ae^{-3t} + Be^{-2t} + \frac{1}{10}[\cos t + \sin t]$$

Write a matlab program to plot the relationship between x if for $0 \le t \le 1$ unit using a step size of 0.01 unit

Soln

- Command Window
- Clear
- Clc
- Close all
- Symst

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x = 0.1*(exp(-3*t) - exp(-2*t) + cos(t) + sin(t))
tn = [0:0.01:15]
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xn = subs (x, tn)
figure (1)
Plot (tn, xn)
grid on
grid minor
axis tight
x label ('time')
y label ('vibrations')

iii) Write the steady state solution of the system in form of $x = k\sin(\omega t + \alpha)$

soln

$x = \frac{1}{10}(e^{-3t} - e^{-2t} + \sin t + \cos t)$

at steady state $\frac{dx}{dt} = 0$

Change in x with time is zero

$\therefore \frac{dx}{dt} = \frac{1}{10}(-3e^{-3t} - e^{-2t} + \cos t - \sin t)$

Note below: The exponentials result zero

$0 = \cos t - \sin t$

$\cos t = \sin t$

$t = 45°$

$x = \frac{1}{10}(\cos 45 + \sin 45) = \frac{\sqrt{2}}{10}$

from sinosodial expression

$A\cos\omega t + B\sin\omega t = k\cos(\omega t - \theta)$

But: $\cos(\omega t - \theta) = \sin(\omega t - \theta + 90°)$

where

$k = \sqrt{A^2 + B^2} = \sqrt{(\frac{1}{10})^2 + (\frac{1}{10})^2}$

$K = \frac{\sqrt{2}}{10}$

$\theta = 0°$ (since it is in the same phase)

Recall $x = k\sin(t + \alpha)$

$\frac{\sqrt{2}}{10} = \frac{\sqrt{2}}{10}\sin(45 + \alpha)$

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$1 = \sin(45+a)$

$45 + a = \sin^{-1}(1)$

$a = 90 - 45$

$= 45° \cong \pi/4$

The steady state seen is

$x = \frac{\sqrt{2}}{10} \sin(t + \pi/4)$

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