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Matric 16/ENG051023

Dept Mechanics Engineering

Course ENG 381

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

$$x = C.F + P.I$$

find C.F

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$$

$$m^2 + 5m + 6 = 0$$

$$m = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$m = \frac{-5 \pm \sqrt{1}}{2}$$

$$m = \frac{-5 \pm 1}{2}$$

$$m = -2 \text{ or } -3$$

C.F is

$$x = Ae^{-3t} + Be^{-2t} \quad \text{--- (I)}$$

P.I

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\frac{d^2x}{dt^2}$$

into eqn (I)

$$-C \cos t - D \sin t + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t = \cos t$$

$$-C \cos t + 5D \cos t + 6C \cos t - D \sin t - 5C \sin t + 6D \sin t = \cos t$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t = \cos t$$

$$-C + 5D + 6C = \cos t \quad |$$

$$5C + 5D = 1$$

$$6D - 5C - D = 0$$

$$5D - 5C$$

$$D = C$$

$$5C + 5D = 1$$

$$5C + 5C = 1$$

$$10C = 1$$

$$C = \frac{1}{10}$$

$$x = C \cos t + D \sin t$$

$$x = \frac{1}{10} \cos t + \frac{1}{10} \sin t \quad \text{--- (II)}$$

When $t = 0$, $x = 0.1$ and $\frac{dx}{dt} = 0$

$$x = C_1 e^{-3t} + P_2 e^{-2t}$$

$$x = A e^{-3t} + B e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t \quad \text{--- (III)}$$

$$0.1 = A e^0 + B e^0 + \frac{1}{10} \cos(0) + \frac{1}{10} \sin(0)$$

$$0.1 = A + B + \frac{1}{10}$$

$$A + B = 0.1 - \frac{1}{10}$$

$$A + B = 0 \quad \text{--- (IV)}$$

Differentiate $x = A e^{-3t} + B e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$

$$\frac{dx}{dt} = -3A e^{-3t} - 2B e^{-2t} - \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

Substitute $\frac{dx}{dt} = 0$ and $t = 0$

$$0 = -3A e^0 - 2B e^0 - \frac{1}{10} \sin(0) + \frac{1}{10} \cos(0)$$

$$0 = -3A - 2B + \frac{1}{10}$$

$$3A + 2B = 1 \quad \text{--- (II)}$$

Simultaneous equation

$$A + B = 0 \quad \times 2$$

$$3A + 2B = 0.1$$

$$2A + 2B = 0$$

$$- 3A + 2B = 0.1$$

$$- A = -0.1$$

$$A = 0.1$$

Substitute $A = 0.1$ into equation II

$$A + B = 0$$

$$0.1 + B = 0$$

$$B = -0.1$$

Substitute A and B into eqn III

$$x = \frac{1}{10} e^{-3t} - \frac{1}{10} e^{-2t} + \frac{1}{10} \cos t + \frac{1}{10} \sin t$$

$$x = \frac{1}{10} (e^{-3t} - e^{-2t} + \cos t + \sin t)$$