

OYEWALE SEMILORE

Engineering Mathematics (Eng 381)

300 level.

MECHANICAL ENGINEERING

16/ENG06/065

OYEWALE SEMILORE  
MECHANICAL ENGINEERING  
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ENGINEERING MATHS.

(1)  $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$

$y = C.F + P.I$

C.F

$$m^2 + 5m + 6 = 0$$
$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$   $b = 5$   $c = 6$

$$m = \frac{-(5) \pm \sqrt{5^2 - (4 \times 1 \times 6)}}{2 \times 1}$$
$$m = \frac{-5 \pm \sqrt{1}}{2}$$

$m = \frac{-5 + 1}{2}$  or  $\frac{-5 - 1}{2}$

$m = -2$  or  $-3$

$m_1 = -2$   $m_2 = -3$

$\therefore x = Ae^{m_1 t} + Be^{m_2 t}$   
 $x = Ae^{-2t} + Be^{-3t}$

P.I  $f(x) = \cos t$ ,  $x = C \sin t + D \cos t$

$$\frac{dy}{dt} = c \cos t - D \sin t$$

$$\frac{d^2y}{dt^2} = -C \sin t - D \cos t$$

$$\therefore \frac{d^2y}{dt^2} + 5 \frac{dy}{dt} + 6y = \cos t$$

$$(-C \sin t - D \cos t) + 5(C \cos t - D \sin t) + 6(C \sin t + D \cos t) = \cos t$$

$$-C \sin t - D \cos t + 5C \cos t + 5D \sin t + 6C \sin t + 6D \cos t = \cos t$$

$$\sin t (-C + 5D + 6C) + \cos t (-D + 5C + 6D) = \cos t$$

By comparing

$$-C + 5D + 6C = 0 \quad \text{--- (i)}$$

$$-D - 5C + 6D = 1 \quad \text{--- (ii)}$$

$$5C + 5D = 0 \quad \text{--- (iii)}$$

$$+5C + 5D = 1 \quad \text{--- (iv)}$$

$$\text{V.O.F} \quad 5C = 5D$$

$$C = D \quad \text{--- (x)}$$

Substituting C for D into (iv)

$$+5D + 5D = 1$$

$$10D = 1$$

$$D = \frac{1}{10}$$

Substitute  $\frac{1}{10}$  for  $D$  into \*

$$\therefore C = \frac{1}{10}$$

$$\therefore X = \frac{1}{10} \sin t + \frac{1}{10} \cos t$$

$$t \quad x = \frac{1}{10} (\sin t + \cos t)$$

$\therefore$  General equation

$$X = A e^{-2t} + B e^{-3t} + \frac{1}{10} (\sin t + \cos t)$$

Given  $t=0$ ,  $x=0$  and  $\frac{dx}{dt} = 0$

$$0 = A e^{-2(0)} + B e^{-3(0)} + \frac{1}{10} (\sin(0) + \cos(0))$$

$$0 = A + B + \frac{1}{10} (0 + 1)$$

$$0 = A + B + \frac{1}{10}$$

$$A + B = 0 \quad \text{--- (*)}$$

$$\frac{dx}{dt} = -2Ae^{-2t} + (-3Be^{-3t}) + \frac{1}{10} (C - \sin t + \cos t)$$

since  $\frac{dx}{dt} = 0$  and  $t=0$

$$0 = -2Ae^{-2(0)} + (-3Be^{-3(0)}) + 0.1(C - 0 + 1)$$

$$-0.1 = -2A - 3B \quad \text{--- (***)}$$

$$-0.1 = -2A - 3B \quad \times 1$$

$$0 = A + B \quad \times 2$$

~~$$0 = -2A - 3B$$~~

$$A = -B$$

~~$$0 = -2A + 2B$$~~

substituting for A into \*\* eqn

$$3(-B) + 2B = 0.1$$

~~$$-B = 0$$~~

$$-B = 0.1$$

~~$$B = 0$$~~

$$B = -0.1$$

$$A = -B$$

Substitute 0 for B into (\*\*)

$$A = 0.1$$

~~$$0 = A + B$$~~

~~$$A = 0$$~~

$$\therefore x = 0.1 + e^{-3t} - 0.1 * e^{-2t} + 0.1 (\cos t + \sin t)$$

$$x = 0.1 (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

$$x = \frac{1}{10} (e^{-3t} - e^{-2t} + \cos t + \sin t)$$

18. Write a steady-state solution of the system in form of  $x = k \sin(t + \phi)$

Answer

From the equation

$$x = A e^{-2x} + B e^{-3x} + \frac{1}{10} (\sin t + \cos t)$$

The equation has a changing or transient part and a steady part.

The transient part denoted as  $e^{k_1 t}$  and  $e^{k_2 t}$

The steady part denoted  $\frac{1}{10} (\sin t + \cos t)$

In trigonometric identities

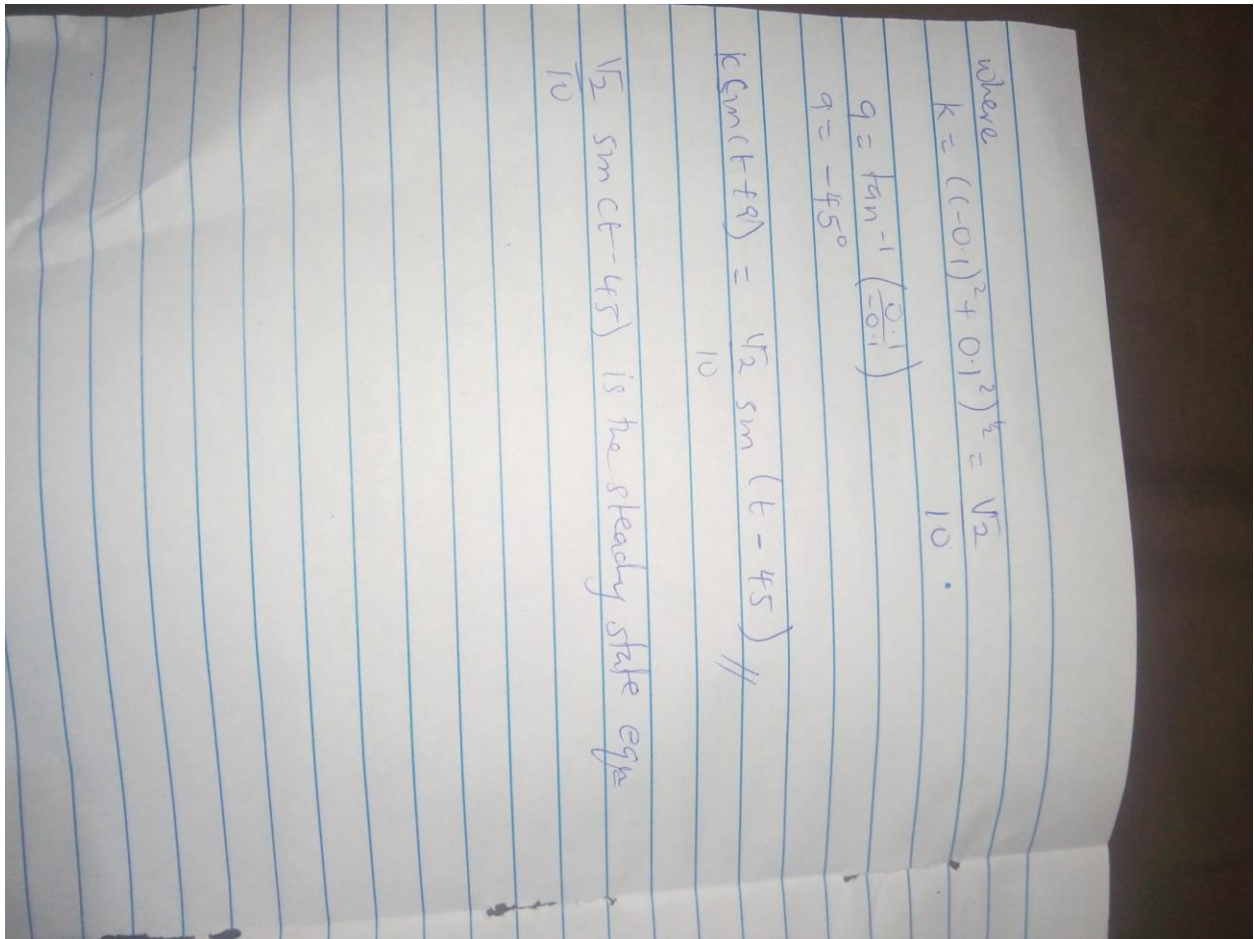
$$A \sin \omega t + B \cos \omega t = C \sin(t + \theta)$$

$$\text{where } \theta = \tan^{-1}(A/B)$$

$$C = (A^2 + B^2)^{1/2}$$

∴ Steady part becomes:

$$\frac{1}{10} \sin t + \frac{1}{10} \cos t = k \sin(t + \phi)$$



MATLAB R2017a

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- Ex 4johnson.m
- Ex 5.m
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- Ex 7.m
- Ex 8.m
- Ex 9.m

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```
1 - commandwindow
2 - clear
3 - clc
4 - close all
5 - syms t
6 - x = (1/10)*(-exp(-2*t) + exp(-3*t) + sin(1*t) + cos(1*t))
7 - ts = (0:0.01:15)
8 - xs = subs(x, t, ts)
9 - figure(1)
10 - plot(ts, xs)
11 - xlabel('Time (seconds)')
12 - ylabel('Vibrations')
13 - grid on
14 - grid minor
15 - axis tight
```

Command Window

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fx >>

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