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DEPT: COMPUTER ENGR

$$\textcircled{1} \frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = \cos t$$

using auxilliary method

$$m^2 + 5m + 6 = 0 \text{ --- auxilliary eqn.}$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{where } a=1, b=5, c=6$$

$$m = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

$$m = \frac{-5 \pm \sqrt{25 - 24}}{2}$$

$$m = \frac{-5 \pm \sqrt{1}}{2}$$

$$\therefore m_1 = \frac{-5 + \sqrt{1}}{2} \quad m_2 = \frac{-5 - \sqrt{1}}{2}$$

$$m_1 = -2, m_2 = -3$$

$$x = Ae^{-2t} + Be^{-3t}$$

For the R.H.S;

$$x = \cos t$$

$$x = C \cos t + D \sin t$$

$$\frac{dx}{dt} = -C \sin t + D \cos t$$

$$\frac{d^2x}{dt^2} = -C \cos t - D \sin t$$

$$\begin{aligned} &+ (-C \cos t - D \sin t) + 5(-C \sin t + D \cos t) + 6(C \cos t + D \sin t) \\ &= \cos t \end{aligned}$$

$$-C \cos t - D \sin t - 5C \sin t + 5D \cos t + 6C \cos t + 6D \sin t$$

$$(-C + 5D + 6C) \cos t + (-D - 5C + 6D) \sin t$$

$$(5C + 5D) \cos t + (-5C + 5D) \sin t = \cos t$$

$$5C + 5D = 1 \text{ --- (1)}$$

$$-5C + 5D = 0 \text{ --- (2)}$$

$$10C + 0 = 1$$

$$\therefore 10C = 1, C = 0.1$$

Substitute C into eqn. (1)

$$5(0.1) + 5D = 1$$

$$0.5 + 5D = 1$$

$$5D = 1 - 0.5$$

$$5D = 0.5$$

$$D = 0.1 \quad \therefore C$$

Complete gen. solution

$$Ae^{-2t} + Be^{-3t} + 0.1 \cos t + 0.1 \sin t$$

$$\text{When } t=0, x=0.1 \quad \& \quad \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = -2Ae^{-2t} + 3Be^{-3t} - 0.1 \sin t + 0.1 \cos t$$

$$dx \text{ when } t=0 \quad \& \quad x=0.1$$

$$0.1 = A + B + 0.1 \text{ --- } x$$

$$0.1 + 0.1 = A + B$$

$$\therefore A + B = 0$$

$$0 = -2A - 3B + 0.1 \text{ --- } \frac{dx}{dt}$$

$$-0.1 = -2A - 3B \text{ --- (3)}$$

$$0 = A + B \text{ --- (4)}$$

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$$-0.1 = -2A - 3B \quad \text{--- (5)}$$

$$0 = -2A + 2B \quad \text{--- (6)}$$

$$-0.1 = 0 - 1B$$

$$-1B = -0.1$$

$$B = \frac{-0.1}{-1}$$

$$B = 0.1$$

Substitute B into eqn. (5)

$$-2A - 3B = -0.1$$

$$-2A - 3(0.1) = -0.1$$

$$-2A - 0.3 = -0.1$$

$$-2A = -0.1 + 0.3$$

$$\frac{-2A}{-2} = \frac{0.2}{-2}$$

$$A = -0.1$$

$$x = -0.1e^{-2t} + 0.1e^{-3t} + 0.1\cos t + 0.1\sin t$$
