Name : Guy Galloway Afuba
Matric No : 16/ENG03/030
Dept : Civil
Course code : ENG 381

$$\frac{d^2x}{dx^2} + 5\frac{dx}{dx} + 6x = \cos t \text{, if } t=0, x=0; r=\frac{du}{dt}=0$$

Solution

Assume $x = Ae^{kt}$

$\frac{dx}{dt} = kAe^{kr} = kx$ $\frac{d^2x}{dt^2} = k^2Ae^{kt} = k^2k$

$k^2 + 5kx + 6x = 0$

divide through by x

$k^2 + 5k + 6 = 0$

$(k+3)(k+2) = 0$

$k = -3$ $k_2 = -2$

$CF : x = Ae^{-3t} + Be^{-2t}$

$x = \cos t$

$x = C\cos t + D\sin t$

$\frac{dx}{dt} = -C\cos t + D\cos t$

$\frac{d^2x}{dt^2} = -C\cos t - D\sin t$

$-C\cos t - D\sin t + 5(-C\sin t + D\cos t) + 6(C\cos t + D\sin t) =$
$\cos t$

$-C\cos t - D\sin t - 5C\sin t + 5D\cos t + 6C\cos t + 6D\sin t = \cos t$

$(-C + 5D + 6C)\cos + (-D - 5C + 6D)\sin t = \cos t$

$(5D + 5D)\cos t + (5D - 5C)\sin t = \cos t$

$5D - 5C = 0$

$5D = 5C$

$D = C$ ---- (1)

$5C + 5D = 1$

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$\cancel{5C = 1 - 5D}$
$\cancel{5C = 1 - 5}$

$5C + 5C = 1$

$10C = 1$

$C = 1/10$

$\therefore D = 1/10$

P.I $\therefore x = 1/10 \cos t + 1/10 \sin t$

$\therefore$ the general solution is $x = Ae^{-3t} + Be^{-2t} + 1/10 \cos t + 1/10 \sin t$

at $t = 0$, $x = 0.1$, $dx/dt = 0$

$x = Ae^{-3t} + Be^{-2t} + 1/10 \cos t + 1/10 \sin t$

$0.1 = Ae^{0} + Be^{0} + 1/10 \cos 0 + 1/10 \sin 0$

$0.1 = A + B + 1/10$

$0.1 = A + B + 0.1$

$A + B = 0.1 - 0.1$

$A + B = 0$ ______ (1)

$\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - 1/10 \sin t + 1/10 \cos t$

$0 = -3Ae^{0} - 2Be^{0} + 1/10$

$-3A - 2B = -0.1$

$3A + 2B = 0.1$ ______ (2)

From eqn (1) $A = -B$ ______ (3)

Substitute eqn (3) into eqn (2)

$\cancel{3A + 2(-B) = 0.1}$

$-3B + 2B = 0.1$

$-B = 0.1$

$B = -0.1$

$\therefore A = 0.1$

$\therefore x = 0.1e^{-3t} + -0.1e^{-2t} + 1/10 \cos t + 1/10 \sin t$

iii) $x = k\sin(t + a)$

To solve the steady-state solution

$$x = \frac{1}{10}\left(\left(-e^{-2t} + e^{-3t}\right) + \left(\cos t + \sin t\right)\right)$$

Transient part of equation

Steady state part of the equation

Using the steady state part of the equation

$$x = \frac{1}{10}(\cos t + \sin t)$$

Note: $\frac{dx}{dt} = 0$ for steady state

$$\frac{dx}{dt} = \frac{1}{10}(-\sin t + \cos t) = 0$$

$$-\sin t + \cos t = 0$$

$$\cos t = \sin t$$

Hence: $t = 45^\circ$ or $\frac{\pi}{4}$

$$x = \frac{1}{10}(\cos 45 + \sin 45)$$

Sinusodial equation: $A\cos wt + B\sin wt = k\cos(wt - \theta)$

but

$$\cos(wt - \theta) = \sin(wt - \theta + 90^\circ)$$

where

$$k = \sqrt{A^2 + B^2} = \sqrt{(1/10)^2 + (1/10)^2} = 0.14$$

Since it is on the same phase, hence: $\theta = 0^\circ$

$$\therefore x = \frac{1}{10}(\cos 45 + \sin 45)$$

$$= 0.14\sin(45 + 90^\circ)$$

$$x = 0.14\sin(45 + 90)$$

$$x = 0.14\sin(90 + 45)$$

in terms of the above equation, hence

$x = 0.14$ or $\frac{\sqrt{2}}{10}$

$a = 45$ or $\frac{\pi}{2}$

$$x = 0.14\sin(t + 45)$$

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